

A Guide to StatFACTS

Investment analysis can be easy. Zephyr has developed StatFACTS to help you understand the plain-English meanings and practical applications of key performance statistics. Moreover, Zephyr has organized the statistics into a logical, useful framework. As Albert Einstein purportedly said, “Everything should be made as simple as possible, but not simpler.”

What Stats Measure

Despite a seemingly overwhelming number of metrics, virtually every statistic falls into one of three categories:

- **Return:** the higher the better
- **Risk:** the lower the better
- **Return vs. Risk Trade-Off:** the higher the better

What Does One Mean By Risk?

Risk can be broken into four broad categories of measurement:

- **Volatility:** volatility risk measures how uncertain or unpredictable an investment’s returns are
- **Benchmark:** benchmark risk measures performance of managers against an appropriate passive index
- **Capital Preservation:** also known as drawdown risk, capital preservation risk is simply the risk of losing money in an investment
- **Tail:** sometimes called “black swan” risk, tail risk is defined as low probability, high impact events

Zephyr StatMAP

As a part of the StatFACTS series, Zephyr also introduces StatMAP. StatMAP offers a way to visualize each statistic in terms of metric versus risk type.

StatMAP		TYPE OF RISK			
		VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
WHAT IS BEING MEASURED	RETURN		<ul style="list-style-type: none"> • Batting Average • Excess Return • Up Capture 		<ul style="list-style-type: none"> • Skewness • Upside Omega
	RISK	<ul style="list-style-type: none"> • Downside Deviation • Standard Deviation 	<ul style="list-style-type: none"> • Beta • Down Capture • R-Squared • Tracking Error 	<ul style="list-style-type: none"> • Maximum Drawdown • Pain Index 	<ul style="list-style-type: none"> • Conditional Value at Risk • Downside Omega • Kurtosis • Value at Risk
	TRADE-OFF	<ul style="list-style-type: none"> • Sharpe Ratio • Sortino Ratio • Zephyr K-Ratio 	<ul style="list-style-type: none"> • Alpha • Information Ratio • Treynor Ratio 	<ul style="list-style-type: none"> • Pain Ratio 	<ul style="list-style-type: none"> • Omega

Zephyr StatFACTS are available for many of the key statistics.

Here, the statistic is placed in the context of the StatMAP

We start with the plain-English definition.

Understanding the context is all-important when it comes to interpreting the numbers. The results are often highly dependent upon the asset class and time frame being analyzed. The back page of StatFACTS contains images, numbers, and text to help you understand each statistic in question.

Alpha

Alpha measures the risk-adjusted added value an active manager adds above and beyond the passive benchmark.

How Is It Useful?
Alpha is often described as a measure of a manager's skill or ability to add value over a passive benchmark. It is important to remember that alpha first adjusts for the degree of market risk undertaken by the manager. Alpha is what remains after the market risk, or beta, is netted out.

What Is a Good Number?
One would want to see a positive value for alpha, and the higher, the better. Positive alpha indicates that after adjusting for market or "systematic" risk, the manager was able to outperform a passive benchmark. Alpha can be generated by superior security selection, over-undervaluing sectors, market timing, or any variety of factors. A positive alpha indicates that those active management decisions paid off.

What Are the Limitations?
Alpha measures returns relative to a market benchmark. A manager can have a respectable alpha (e.g. an alpha of +3.5), but the overall returns of the manager could be negative if the benchmark itself had negative performance.

What Does the Graph Show Me?
Below are two managers, an aggressive manager in red and a conservative manager in blue. Looking only at excess return over the benchmark, the aggressive manager looks preferable to the conservative manager. However, the key to the calculation of alpha is the amount of beta-risk the manager has undertaken. In the graph below, the aggressive manager's beta (1.4) is double that of the conservative manager's beta (0.7).

Because the aggressive manager took on so much risk, the excess return should be much higher. The negative alpha indicates that the aggressive manager was not adequately compensated for the high level of risk. The conservative manager was able to outperform the benchmark while maintaining a lower level of risk, resulting in a positive alpha.

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Alpha

What Are Typical Values?
The table to the right displays the ranges of 10-year alphas across six asset classes. Peer groups of separately managed account composites are compared to their relevant benchmarks. The data here suggests that the median manager typically has an alpha near zero. Some funds do better than the benchmark, and some worse, but the distribution is centered around zero. The range of alphas is widest in small cap US stocks and international stocks, which are often perceived as more inefficient asset classes where active managers tend to take larger bets.

Alpha Ranks in the Universe	Large Cap US	Small Cap US	International US	Emerging Mkts	Govt Corp	HY Bond
5th Percentile	2.29%	3.62%	4.55%	4.25%	0.89%	3.67%
25th Percentile	0.63%	1.40%	1.21%	1.40%	0.39%	0.43%
Median	-0.19%	-0.18%	-0.34%	-0.78%	0.07%	-0.47%
75th Percentile	-0.59%	-0.59%	-1.50%	-1.73%	-0.24%	-1.20%
95th Percentile	-1.72%	-2.49%	-3.24%	-2.81%	-0.91%	-2.19%

Related Metrics

Beta: the sensitivity of a manager to a benchmark

Information Ratio: a manager's added value and consistency of added value

Excess Return: the difference between a manager's returns and the benchmark's returns

Math Corner

The simpler, standard definition of alpha is to treat a manager's total returns as a combination of two components, a portion that is a function of market movements and a portion that is unique to the individual manager. Rearranging the terms, alpha can be expressed as:

$$\text{Alpha} = r_{mgr} - [\beta * r_{BM}]$$

Another common version of alpha is known as Jensen's alpha or cash-adjusted alpha. This version first subtracts out a risk-free rate from both the manager returns and the benchmark returns before proceeding with the standard alpha calculation. Jensen's alpha is more in-synch with the Capital Asset Pricing Model (CAPM). It is written:

$$\text{Jensen's Alpha} = [r_{mgr} - r_f] - \beta[r_{BM} - r_f]$$

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Digging deeper into each metric, these three sections address the questions, "How is it useful?," "What is a good number?" and "What are the limitations?"

A picture is worth a thousand words. Graphs from Zephyr StyleADVISOR illustrate the Information captured by each statistic.

StatFACTS are built to be simple and easy to understand. However, for those who want to take it to the next level, we offer the "Math Corner." Here you'll see the mathematical formula and description of how the statistic is calculated.

A Quick Reference to StatFACTS

Zephyr has developed this reference guide to our StatFACTS series to relay key Information about each statistic at-a-glance.

	How Is It Classified?	What Type of Risk?	What Is It?	What Should I Be Looking For?	What Are The Limitations?
Alpha	Return-vs.-Risk Trade-off	Benchmark	Alpha measures the risk-adjusted added value an active manager adds above and beyond the passive benchmark.	Alphas should be positive. A negative alpha suggests the manager failed to add value over the benchmark on a risk-adjusted basis.	It is important to use an appropriate benchmark. It is possible to have a positive alpha, but still lose money, if the benchmark is down.
Batting Average	Return	Benchmark	An indicator of consistency, batting average measures the percentage of time an active manager outperformed the benchmark.	Ideally, a manager should outperform its benchmark at least half of the time. The higher the batting average, the more frequent the outperformance.	Batting average does not take risk or the scale of outperformance into account.
Beta	Risk	Benchmark	Beta measures the sensitivity of a manager's movements in an underlying benchmark.	Conservative investors prefer a beta less than 1.0, suggesting the investment moves less than the market. Aggressive investors prefer a beta greater than 1.0, which are more sensitive to market movements.	Beta does not distinguish between up markets and down markets. An investor would prefer to be up more than the market when markets are up, but down less when markets are down. Beta does not differentiate.
Conditional Value at Risk	Risk	Tail	Conditional Value at Risk (CVaR) quantifies the scale of expected losses once the Value at Risk (VaR) breakpoint has been breached.	Like VaR, the smaller the value for CVaR, the better. One would hope that the VaR breakpoint is rarely penetrated. When VaR is surpassed, one would hope it is not exceeded by a significant amount.	Like VaR, CVaR does not represent the maximum one could potentially lose in an investment. With any investment, the maximum possible loss is 100%.
Down Capture	Risk	Benchmark	Down capture measures the percentage of market losses endured by a manager when markets are down.	Down capture should be less than 100%, meaning a manager experiences less than the full market downswing.	The manager must have a significant number of down periods to obtain a useful down capture. It should be used in conjunction with up capture.
Downside Deviation	Risk	Volatility	Downside deviation is a risk statistic measuring volatility. It is a variation of standard deviation that focuses only upon the "bad" volatility.	Generally, the lower the better. A manager's downside deviation should be lower than the index or lower than the universe's average.	Downside deviation needs enough "bad" data points to be useful.
Downside Omega	Risk	Tail	Downside omega measures the count and scale of returns below a minimum acceptable return (MAR).	The lower the downside omega, the fewer and/ or less extreme the "bad" observations are short of the minimum acceptable return (MAR). The lowest possible value would be 0.0.	Most investments that have low downside omega also have low upside omega. It is rare to find an investment with limited downside risk, but a lot of upside potential.
Excess Return	Return	Benchmark	The simplest of the benchmark-relative statistics, excess return measures the difference between the manager return and the benchmark return.	One would want the excess return to be positive, indicating the manager outperformed its benchmark.	Excess return does not take risk into account and is often confused with alpha, which does account for the level of risk.
Information Ratio	Return-vs.-Risk Trade-off	Benchmark	A benchmark-relative, return-vs.-risk metric, the Information ratio measures the excess return against the benchmark divided by tracking error, where tracking error is a measure of consistency.	Information ratios should be positive. A good Information ratio is typically in the 0.40-0.60 range. It is rare to see active managers with Information ratios greater than 1.00.	It is important to use an appropriate benchmark. It is possible to have a positive Information ratio, but still lose money if the benchmark is down.
Kurtosis	Risk	Tail	Kurtosis identifies where the volatility risk came from in a distribution of returns. Kurtosis improves one's understanding of volatility risk.	Generally, investors like to see kurtosis numbers close to zero, or even negative. The larger the kurtosis, the more of an investment's risk lies in the tails of the distribution.	Kurtosis must be used in conjunction with standard deviation. Kurtosis as a standalone metric is not very meaningful.
Maximum Drawdown	Risk	Capital Preservation	A risk metric indicating capital preservation, the maximum drawdown measures the peak-to-trough loss of an investment.	The smaller the maximum drawdown, the better. A maximum drawdown of 0% indicates an investment never lost money. One should keep in mind the type of investment and the time period analyzed to understand if a maximum drawdown is reasonable.	Maximum drawdown only measures one dimension (the drawdown) and one event (the maximum one). It does not indicate if the investment recovered its losses, the time to recovery, or anything about the smaller drawdowns.
Omega	Return-vs.-Risk Trade-off	Tail	Omega compares upside gains against downside risks. Omega represents the count and scale of returns above a breakpoint versus the count and scale of observations below a breakpoint.	The larger the omega, the better. However, one needs to compare the omega versus a benchmark or peer group in order to understand what would qualify as being good or bad.	Omega compares upside versus downside in a single metric, so it is difficult to interpret return and risk independently. In order to separate return from risk, use upside omega and downside omega, respectively.

	How Is It Classified?	What Type of Risk?	What Is It?	What Should I Be Looking For?	What Are The Limitations?
Pain Index	Risk	Capital Preservation	A proprietary risk metric, the pain index quantifies the capital preservation tendencies of a manager or index. It measures the depth, duration, and frequency of periods of losses.	The lower the pain index, the better. A pain index of 0% indicates the investment has never lost value. A pain index should be compared against a benchmark or peer group in order to understand context.	Pain index does not take in to account the periods of gains. It is only focused on periods of loss.
Pain Ratio	Return-vs.-Risk Trade-off	Capital Preservation	A proprietary return-vs.-risk trade-off metric, the pain ratio compares the added value over the risk-free rate against the depth, duration, and frequency of losses.	The higher the pain ratio the better. A high pain ratio indicates, 1) a high risk premium over the risk free rate, 2) very little losses, or 3) a combination of both. One should compare an investment's pain ratio to a benchmark or universe.	A good pain ratio is a large number, which can be confusing. If one understands return is divided by risk, it follows the ratio should be large. However, without understanding the math, one might incorrectly assume a low pain ratio is better.
R-Squared	Risk	Benchmark	R-squared represents the "goodness of fit" of a manager to its benchmark. R-squared is the percentage of variation in a manager's returns explained by the benchmark's returns.	An investor who believes it is difficult for active managers to outperform a passive benchmark would likely prefer a high R-squared. Alternatively, an investor who believes in active management would prefer a lower R-squared.	R-squared doesn't inform us whether or not performance has been good or bad. It is simply used to determine the "goodness of fit" of a manager to its benchmark.
Sharpe Ratio	Return-vs.-Risk Trade-off	Volatility	The most famous return-vs.-risk measurement, the Sharpe ratio represents the added value over the risk-free rate per unit of volatility risk.	Generally, the higher the Sharpe ratio, the better. A manager's Sharpe ratio should be higher than the index or higher than the universe's average.	Sharpe ratio has the same limitations as standard deviation, since it is half the equation. Sharpe ratios can be negative.
Skewness	Return	Tail	Skewness measures to what direction and degree a set of returns is tilted, or "skewed," by its extreme outlier occurrences.	Generally speaking, investors prefer a positive skewness rather than a negative skewness. However, in the real world, it is difficult to find an investment with a positive skew.	Skewness must be used in conjunction with return. While skewness tells us the impact of the tail events, it is important to know where the distribution is centered.
Sortino Ratio	Return-vs.-Risk Trade-off	Volatility	A variation of the Sharpe ratio, the Sortino ratio is a return-vs.-risk trade-off metric that uses downside deviation as its measure of risk.	The larger the Sortino ratio, the better. One must compare a manager's Sortino ratio to an index or peer group in order to understand whether or not a Sortino ratio is good or bad. It is also useful to keep in mind the time period being analyzed.	There must be enough downside events for the Sortino ratio to be valid. While a lack of downside events might be good for the investor, they are necessary in order to have statistically significant calculation.
Standard Deviation	Risk	Volatility	Standard deviation measures how closely returns track their long term average. Standard deviation measures volatility risk.	Generally, the lower the standard deviation, the better. A manager's standard deviation should be lower than the index or lower than the universe's average.	Standard deviation punishes upside and downside risk equally. It does not take into account the timing of returns.
Tracking Error	Risk	Benchmark	Also known as the standard deviation of excess returns, tracking error measures how consistently a manager outperforms or underperforms the benchmark.	An investor who likes managers to stick to a well-defined mandate would likely prefer a low tracking error. Alternatively, an investor who prefers managers who are free to pursue a more "active" strategy would prefer a higher tracking error.	Tracking error does not measure whether or not the manager's active management bets have been profitable. It is the standard deviation of excess return, not whether or not the excess return has been positive or negative.
Treynor Ratio	Return-vs.-Risk Trade-off	Benchmark	A return-vs.-risk metric, the Treynor ratio measures the added value per unit of market risk, with beta defined as risk.	The larger the Treynor ratio, the better, but one should compare a manager's Treynor ratio to an index or peer group to understand what is good or bad.	The right benchmark must be used in Treynor ratio. A wrong benchmark might yield a low Beta, like 0.0 or 0.1, and then the Treynor ratios will appear deceptively large.
Up Capture	Return	Benchmark	Up capture measures the percentage of market gains captured by a manager when markets are up.	Ideally, up capture will be greater than 100%, meaning the manager does better than the market when markets are up. The larger the up capture, the better.	Up capture should be used in conjunction with down capture to understand the risks.
Upside Omega	Return	Tail	Upside omega measures the count and scale of returns above a minimum acceptable return (MAR).	The higher the upside omega, the better. The higher the upside omega, the more frequent and/or the more extreme are the "good" observations above the MAR.	Most investments with a high upside omega also have a high downside omega. It is rare to find an investment with a lot of upside potential, but limited downside risk.
Value at Risk	Risk	Tail	Value at Risk (VaR) quantifies the amount of expected loss under rare-but-extreme market conditions.	Since VaR is a risk metric measuring loss, the smaller the VaR, the better. Ideally the VaR would be 0.0%, but no investment carries zero risk.	VaR does not represent the maximum amount one can possibly lose. The most one could potentially lose is 100% of an investment. VaR is not an absolute number, but a breakpoint that is exceeded only under extreme conditions.
Zephyr K-Ratio	Return-vs.-Risk Trade-off	Volatility	A return-vs.-risk statistic, the Zephyr K-Ratio measures the rate at which wealth is created and the consistency of the path of wealth creation.	The higher the Zephyr K-Ratio, the better, indicating, 1) a rapid increase in wealth, 2) consistency of wealth creation, or 3) both. One should compare Zephyr K-Ratios against indices or universes to understand what is good or bad.	Zephyr K-Ratio is calculated by using advanced statistical techniques, which might be difficult for a layman to understand.

Alpha

Alpha measures the risk-adjusted added value an active manager adds above and beyond the passive benchmark.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				
TRADE-OFF		ALPHA		

How Is it Useful?

Alpha is often described as a measure of a manager’s skill or ability to add value over a passive benchmark. It is important to remember that alpha first adjusts for the degree of market risk undertaken by the manager. Alpha is what remains after the market risk, or beta, is netted out.

What Is a Good Number?

One would want to see a positive value for alpha, and the higher, the better. Positive alpha indicates that after adjusting for market or “systematic” risk, the manager was able to outperform a passive benchmark. Alpha can be generated by superior security selection, over/underweighting sectors, market timing, or any variety of factors. A positive alpha indicates that those active management decisions paid off.

It is entirely possible that a manager could outperform the benchmark and have negative alpha. If the manager undertook excess risks and only generated marginal outperformance versus the benchmark, the manager could exhibit negative alpha.

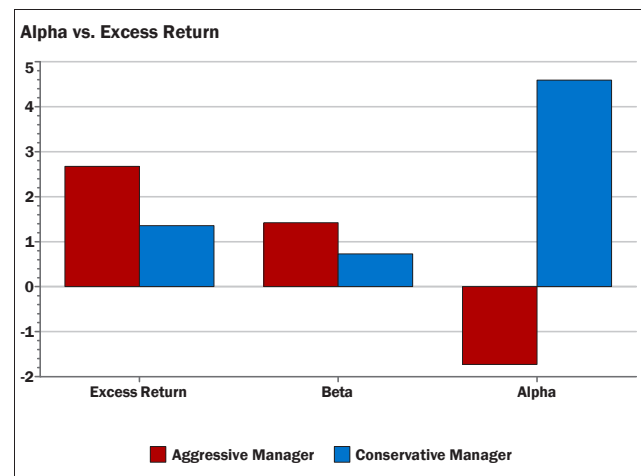
What Are the Limitations?

Alpha measures returns relative to a market benchmark. A manager can have a respectable alpha (e.g. an alpha of +3.5), but the overall returns of the manager could be negative if the benchmark itself had negative performance.

What Does the Graph Show Me?

Below are two managers, an aggressive manager in red and a conservative manager in blue. Looking only at excess return over the benchmark, the aggressive manager looks preferable to the conservative manager. However, the key to the calculation of alpha is the amount of beta-risk the manager has undertaken. In the graph below, the aggressive manager’s beta (1.4) is double that of the conservative manager’s beta (0.7).

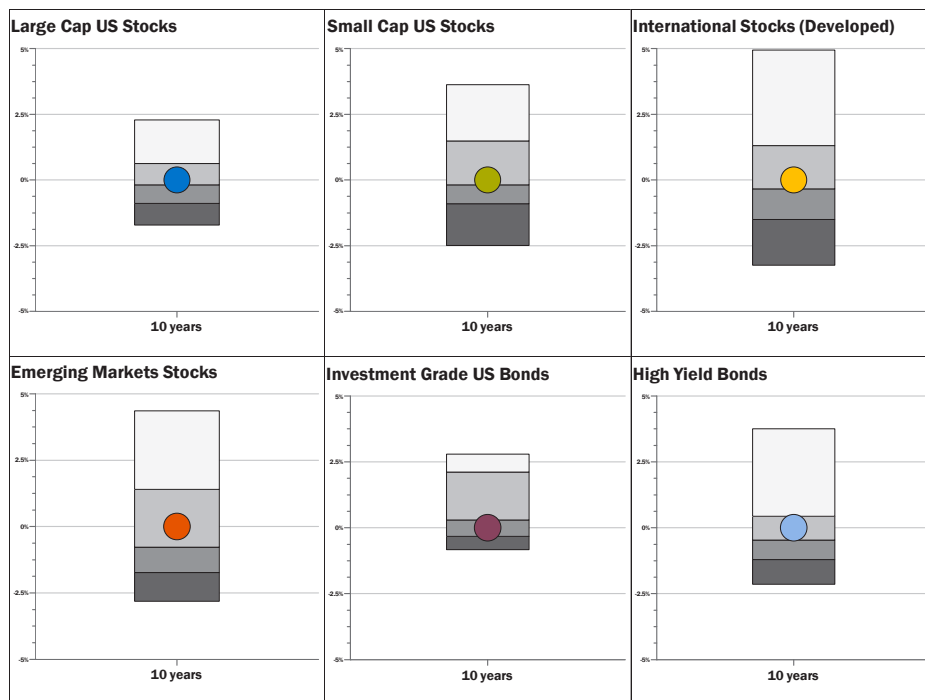
Because the aggressive manager took on so much risk, the excess return should be much higher. The negative alpha indicates that the aggressive manager was not adequately compensated for the high level of risk. The conservative manager was able to outperform the benchmark while maintaining a lower level of risk, resulting in a positive alpha.



Alpha

What Are Typical Values?

The table to the right displays the ranges of 10-year alphas across six asset classes. Peer groups of separately managed account composites are compared to their relevant benchmarks. The data here suggests that the median manager typically has an alpha near zero. Some funds do better than the benchmark, and some worse, but the distribution is centered around zero. The range of alphas is widest in small cap US stocks and international stocks, which are often perceived as more inefficient asset classes where active managers tend to take larger bets.



January 2003 - December 2012 • Symbol = Benchmark Index

Alpha Funds In the Universe	Large Cap 230	Small Cap 94	International 325	Emerging 64	Gov/Corp 293	HY Bond 96
5th Percentile	2.29%	3.62%	4.95%	4.35%	0.89%	3.76%
25th Percentile	0.63%	1.49%	1.31%	1.40%	0.30%	0.43%
Median	-0.19%	-0.18%	-0.34%	-0.78%	0.03%	-0.47%
75th Percentile	-0.90%	-0.90%	-1.50%	-1.73%	-0.26%	-1.20%
95th Percentile	-1.72%	-2.49%	-3.24%	-2.81%	-0.91%	-2.15%

Related Metrics

Beta: the sensitivity of a manager to a benchmark

Information Ratio: a manager's added value and consistency of added value

Excess Return: the difference between a manager's returns and the benchmark's returns

Math Corner

The simpler, standard definition of alpha is to treat a manager's total returns as a combination of two components, a portion that is a function of market movements and a portion that is unique to the individual manager. Rearranging the terms, alpha can be expressed as:

$$\text{Alpha} = r_{mgr} - [\beta * r_{BM}]$$

Another common version of alpha is known as Jensen's alpha or cash-adjusted alpha. This version first subtracts out a risk-free rate from both the manager returns and the benchmark returns before proceeding with the standard alpha calculation. Jensen's alpha is more in-sync with the Capital Asset Pricing Model (CAPM). It is written:

$$\text{Jensen's Alpha} = [r_{mgr} - r_c] - \beta[r_{BM} - r_c]$$

Batting Average

An indicator of consistency, batting average measures the percentage of time an active manager outperformed the benchmark.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN		BATTING AVERAGE		
RISK				
TRADE-OFF				

How Is it Useful?

Batting average is conceptually easy to understand. It is simply the percentage of periods when the manager outperformed the benchmark. The higher the batting average, the more consistent the outperformance.

What Is a Good Number?

The higher the batting average, the better. The highest number possible would be 100%, meaning the manager outperformed the benchmark every single period. On the opposite end of the spectrum would be a batting average of 0%, attainable only if the manager never once managed to outperform the benchmark. Generally speaking, a batting average of 50% is used as a minimum threshold for success.

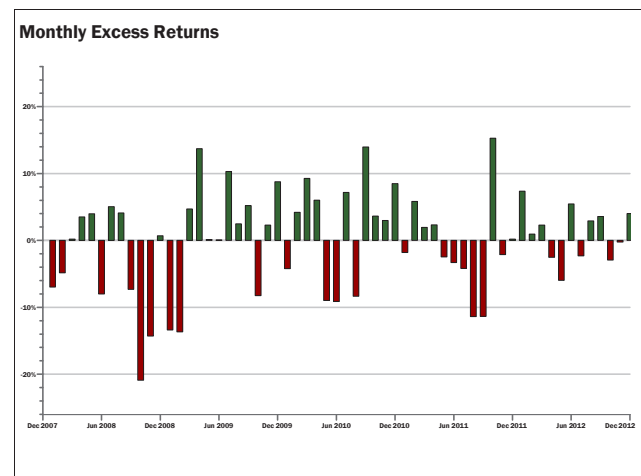
What Are the Limitations?

Batting average has two limitations. First, batting average focuses only on returns and does not take into consideration the amount of risk undertaken by the manager to achieve those returns. Second, batting average does not take into account the scale of the outperformance. A manager might outperform the benchmark by, say, 0.1% for nine months, but in the 10th month, fall short of the benchmark by 5.0%. In such a case, the batting average would be 90%, but the manager would have dramatically underperformed the benchmark.

What Does the Graph Show Me?

The below graph illustrates the monthly outperformance of a manager versus the benchmark. The green bars represent outperformance, and the red bars represent periods of underperformance. The batting average is simply the number of green bars as a percentage of the total number of bars.

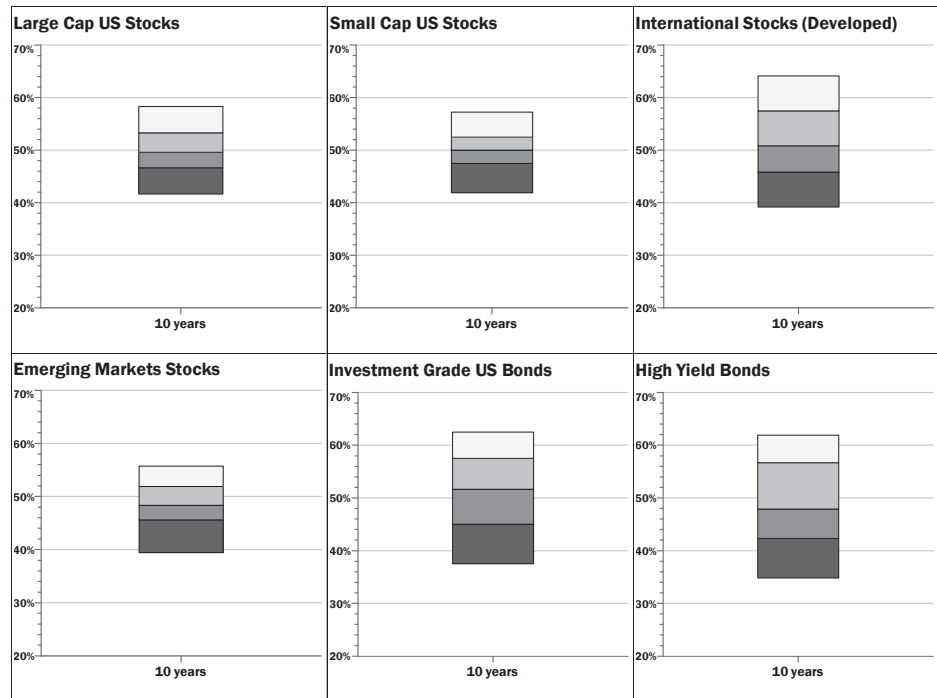
The graph below also illustrates the shortcoming of batting average. The batting average does not take into account the scale of the outperformance or underperformance. Obviously, one would hope to see large green bars and small red bars. However, batting average only measures the count of outperformance periods.



Batting Average

What Are Typical Values?

To the right are ranges of 10-year batting averages for universes of separately managed account composites covering six asset classes. The data shows us that many managers across all six asset classes struggle to outperform their benchmark more than half the time. Even the best managers in the fifth percentile have batting averages that top out in the 55%-60% range. Over long periods of time, even the most successful managers struggle to beat the benchmark three out of every five months.



January 2003 - December 2012

Batting Average Funds In the Universe	Large Cap 230	Small Cap 94	International 325	Emerging 64	Gov/Corp 293	HY Bond 96
5th Percentile	58.33%	57.25%	61.67%	55.71%	65.33%	61.88%
25th Percentile	53.33%	52.50%	54.17%	51.88%	57.50%	56.67%
Median	49.58%	50.00%	48.33%	48.33%	50.83%	47.92%
75th Percentile	46.67%	47.50%	45.00%	45.63%	42.50%	42.29%
95th Percentile	41.67%	41.92%	39.17%	39.42%	34.17%	34.79%

Related Metrics

Excess Return: the difference between a manager's returns and the benchmark's returns

Tracking Error: the standard deviation of excess returns of a manager versus its benchmark

Information Ratio: a manager's added value and consistency of added value

Math Corner

The calculation for batting average is quite simple. Its relative simplicity is both its strength and weakness. It is easy to understand, but limited in what it tells you.

$$\text{Batting Average} = \frac{(\# \text{ of periods where } r_i > BM_i)}{\text{Total \# of periods}}$$

Beta

Beta measures the sensitivity of the manager to movements in an underlying benchmark.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK		BETA		
TRADE-OFF				

How Is it Useful?

Beta answers the question, “When markets go up or down, does the manager typically go up or down more than the market or less than the market?” This is what is meant by market sensitivity. Beta is also used to quantify market risk, sometimes known as “systematic risk”.

What Is a Good Number?

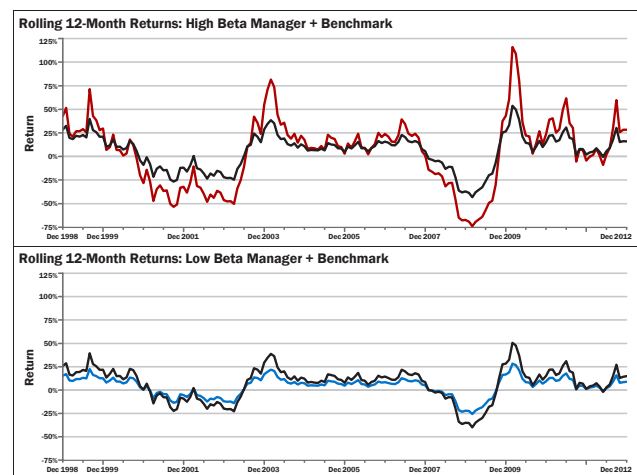
The starting reference point for beta is 1.0. If a manager has a beta of 1.0 to the benchmark, the manager doesn't tend to go up more or less than the market when the market moves. A more conservative manager who tends to trail in up markets, but has downside protection in falling markets, would have a beta less than 1.0. An aggressive manager who did very well in up markets, but tended to lose more in down markets, would have a beta greater than 1.0. Therefore, the definition of “good” depends upon whether the investor is conservative or aggressive.

What Are the Limitations?

Beta does not distinguish between up and down markets. Ideally one would want a manager who was up more than the benchmark when markets were up, but down less than the benchmark during down markets. However, beta only measures overall sensitivity and does not differentiate between up and down markets.

What Do the Graphs Show Me?

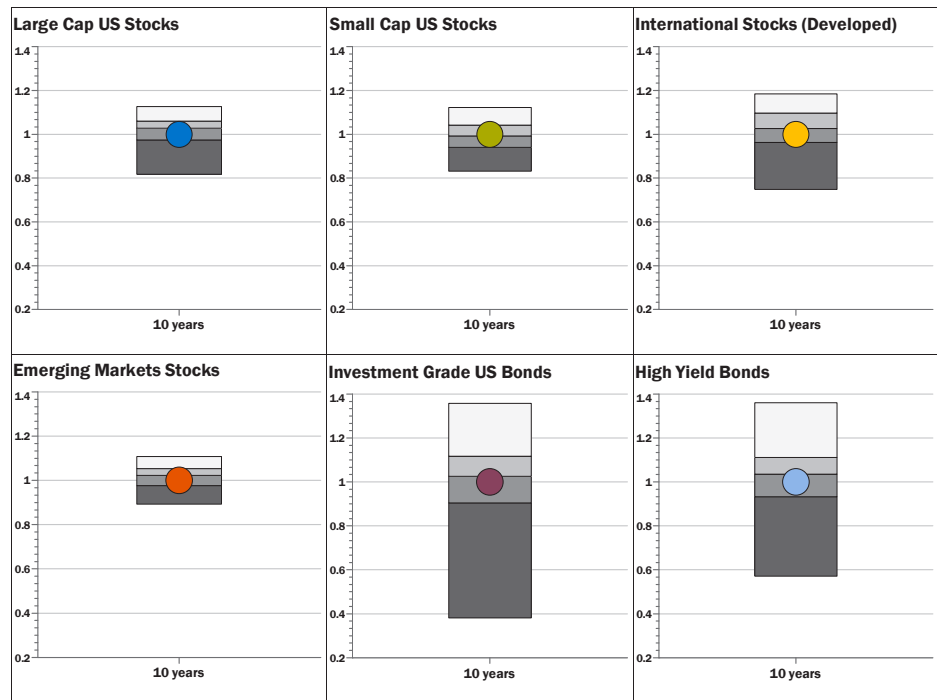
Both graphs below share the same starting point, with rolling benchmark returns in black. As the black line for the benchmark oscillates between periods of gains and periods of losses, the two managers react differently. In the upper graph, the manager in red tends to do very well when markets are up, but when markets are down, the manager performs worse than the benchmark. The red manager is sensitive to market movements and would exhibit a beta higher than 1.0. On the other hand, the blue manager lags the benchmark when the market is up, but tends to offer downside protection when the market is down. The blue manager has a beta less than 1.0 and isn't as sensitive to market movements.



Beta

What Are Typical Values?

The Information to the right shows the typical range of mutual fund betas relative to the appropriate benchmark. Across most asset classes, the middle range of the distribution (funds in between the 25th and 75th percentiles) has betas close to 1.0. This means that most funds aren't overly or less sensitive to market movements. The betas are more interesting in the tails. The ranges seem to be tilted towards lower betas, meaning that there are more conservative managers than aggressive managers.



January 2003 - December 2012 • Symbol = Benchmark Index

Beta Funds In the Universe	Large Cap 230	Small Cap 94	International 325	Emerging 64	Gov/Corp 293	HY Bond 96
5th Percentile	1.13	1.12	1.18	1.11	1.36	1.36
25th Percentile	1.06	1.04	1.10	1.05	1.12	1.11
Median	1.03	0.99	1.03	1.02	1.03	1.04
75th Percentile	0.97	0.94	0.96	0.98	0.90	0.93
95th Percentile	0.82	0.83	0.75	0.89	0.38	0.57

Related Metrics

Alpha: a measure of “manager skill,” adjusted for the level of market risk

Up Capture: the percentage of the benchmark’s positive returns experienced by the manager

Down Capture: the percentage of the benchmark’s negative returns experienced by the manager

Treynor Ratio: the trade-off of return per unit of beta, or market-sensitivity risk

Math Corner

The numerator of beta measures how the manager return moves relative to the benchmark movements. The denominator scales the results of the numerator so that the point of reference of beta is 1.0.

$$Beta = \frac{\sum_{(i-1)}^n (r_i - \bar{r}) * (BM_i - \overline{BM})}{\sum_{(i-1)}^n (BM_i - \overline{BM})^2}$$

Conditional Value at Risk

A tail risk metric, Conditional Value at Risk (CVaR), quantifies the scale of expected losses once the Value at Risk (VaR) breakpoint has been breached.

StatMAP				
	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				CONDITIONAL VALUE AT RISK
TRADE-OFF				

How Is it Useful?

Conditional Value at Risk attempts to address some of the shortcomings of Value at Risk (VaR). VaR is defined as a breakpoint that is breached only under extreme conditions. However, VaR does not describe what happens beyond that breakpoint. CVaR does. It is the average of the returns that fall beyond the VaR cut-off. CVaR is a more pessimistic measure of tail risk than VaR.

What Is a Good Number?

Like VaR, the smaller the value for CVaR, the better. One would hope that the VaR breakpoint is rarely penetrated. When VaR is surpassed, one would hope it is not exceeded by a significant amount. Therefore, CVaR is most useful when viewed in conjunction with VaR.

What Are the Limitations?

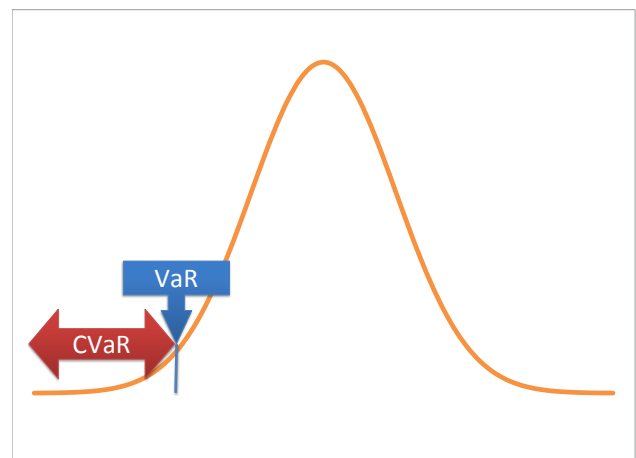
Like VaR, CVaR does not represent the maximum one could potentially lose in an investment. With any investment, the maximum possible loss is 100%.

Also, CVaR is focused solely on risk. It does not account for the upside potential of an investment. Those investments with the largest downside tail risk are often those with the greatest upside potential.

What Do the Graphs Show Me?

The graph below illustrates an idealized distribution curve of returns. Most of the time, an investment's returns occur near the center, or "peak," of the distribution. When markets are doing very well, the returns will fall to the far right of the curve. However, at other times, the returns will fall to the left or far-left of the distribution.

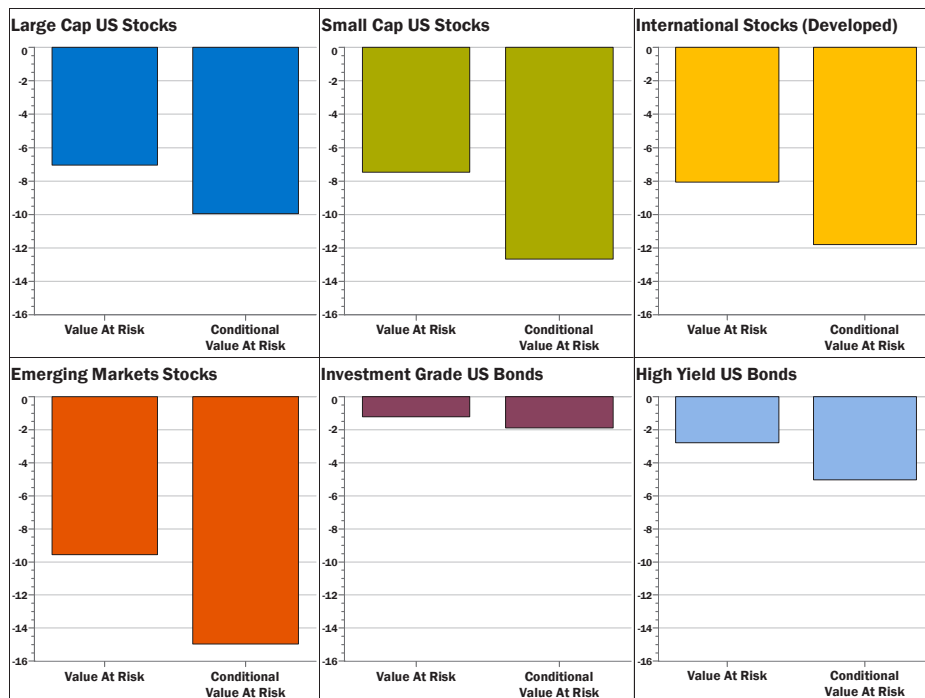
The point marked as VaR represents a breakpoint that is rarely expected to be surpassed. CVaR explores what happens on those occasions when the VaR cut-off is breached. CVaR is the average of the extreme losses in the "tail" of the distribution.



Conditional Value at Risk

What Are Typical Values?

CVaR is best analyzed in conjunction with its close relative, VaR. VaR is a breakpoint, CVaR is what happens when VaR is breached. With large cap US stocks and investment grade US bonds, when returns fall past the VaR breakpoint, they don't tend to exceed the VaR by a significant amount. In contrast, more volatile asset classes like small cap US stocks, emerging markets stocks, and high yield US bonds tend to exhibit CVaRs of magnitudes up to 50% greater than VaRs. In plain-English, this means that when things are bad, they are really bad.



January 1986 - December 2012

	Value at Risk	Conditional Value at Risk
Large Cap US Stocks	-7.05%	-9.95%
Small Cap US Stocks	-7.48%	-12.66%
International Stocks (Developed)	-8.06%	-11.79%
Emerging Markets Stocks	-9.57%	-14.96%
Investment Grade US Bonds	-1.21%	-1.89%
High Yield US Bonds	-2.79%	-5.04%

Related Metrics

Value at Risk (VaR): the amount of expected loss under rare-but-extreme market conditions

Pain Index: the depth, duration, and frequency of losses

Maximum Drawdown: the largest peak-to-trough losses

Downside Omega: the count and scale of returns below a breakpoint

Math Corner

The very name, Conditional Value at Risk, indicates how it is calculated. CVaR values are conditional to the calculation of VaR itself. Therefore, all of the decisions that go into the calculation of VaR will also impact CVaR. The shape of the distribution, the cut-off level used, the periodicity of the data, and assumptions about volatility stochasticity all set the value known as VaR.

Once the VaR has been established, calculating CVaR is trivial. It is simply the average of those values that fall beyond the VaR:

$$CVaR = \frac{1}{1-c} \int_{-1}^{VaR} xp(x)dx$$

Where $p(x)dx$ term is the probability density of getting a return with value "x" and "c" is the cut-off point along the distribution curve where one sets the VaR breakpoint.

Downside Deviation

Downside deviation is a risk statistic measuring volatility. It is a variation of standard deviation that focuses only upon the “bad” volatility.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK	DOWNSIDE DEVIATION			
TRADE-OFF				

How Is it Useful?

Downside deviation addresses a shortcoming of standard deviation, which makes no distinction between the “good,” or upside deviations, and the “bad,” or downside deviations. Both upside and downside deviations have an equal influence on the calculation of standard deviation. Downside deviation seeks to remedy this by ignoring all of the “good” observations and by instead focusing on the “bad” returns.

What Is a Good Number?

Like most other risk metrics, the lower the number, the better. A value of zero would be the best possible value. However, it is important to understand the number in the proper context. One would need to look at the downside deviation of a relevant benchmark or an appropriate peer group in order to get a good feel for the downside deviation.

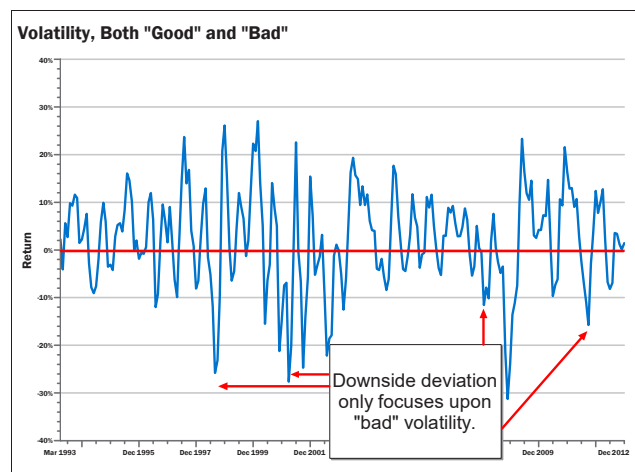
What Are the Limitations?

In order for a downside deviation measure to be useful, there must be enough “bad” observations for the calculation to be statistically significant. While an investor might feel that lacking “bad” events in a data stream is actually positive, the usefulness of the metric would be limited if that was the case.

What Does the Graph Show Me?

The below graph shows a volatile series of returns used for the calculation of both standard deviation and downside deviation. Standard deviation incorporates all of the datapoints in the series. With downside deviation, the “good” months are excluded, and only the “bad” months are counted.

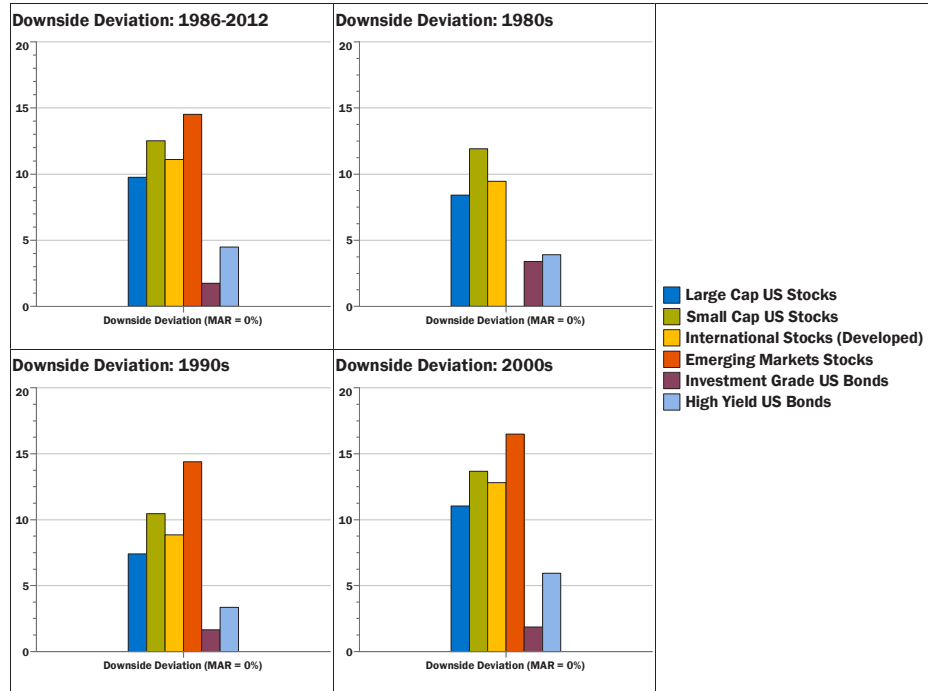
There are several different ways in which one can define what counts as a “bad” observation with downside deviation. One might consider any negative return to be a “bad” observation. Alternatively, one could set the breakpoint as falling short of the risk-free rate. Another variation would be to consider any return that is less than the long-term average to be “bad”.



Downside Deviation

What Are Typical Values?

In the table to the right, the downside deviation was calculated with a minimum acceptable return (MAR) of 0.0%. In other words, only monthly returns less than zero would be counted in the calculation of downside deviation. Two takeaways are apparent. Equity asset classes have higher downside deviations, as returns more frequently fall short of the 0.0% MAR. Also, this happened more frequently in the 2000s than in the 1980s and 1990s, so downside deviations tend to be higher in the most recent decade.



January 1986 - December 2012

Downside Deviation	1980s	1990s	2000s	Common 1/86 - 12/12
Large Cap US Stocks	8.42%	7.41%	11.05%	9.76%
Small Cap US Stocks	11.92%	10.46%	13.68%	12.53%
International Stocks (Developed)	9.45%	8.86%	12.82%	11.12%
Emerging Markets Stocks	N/A	14.39%	16.49%	14.51%
Investment Grade US Bonds	3.39%	1.66%	1.86%	1.74%
High Yield US Bonds	3.92%	3.35%	5.94%	4.49%

Related Metrics

Sortino Ratio: the trade-off of return per unit of downside volatility risk

Standard Deviation: the degree to which individual returns diverge from the average return

Upside Deviation: the amount of “good” volatility risk

Math Corner

The most important variable in the equation for downside deviation is the definition for what counts as being a “bad” observation. Denoted as “c” below, only the returns less than “c” are included in the calculation for downside deviation. Frequently used values for “c” are the risk-free rate, a hard-target value like 0%, or the mean return of the return series itself.

$$\text{Downside Dev} (r_1, \dots, r_n) = \sqrt{\frac{1}{n} \sum_{r_i < c} (r_i - c)^2}$$

Excess Return

The simplest of the benchmark-relative statistics, excess return measures the difference between the manager return and the benchmark return.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN		EXCESS RETURN		
RISK				
TRADE-OFF				

How Is it Useful?

Excess return is simple to understand and doesn't require any sophisticated statistical knowledge. One calculates excess return using nothing more complicated than subtraction.

What Is a Good Number?

One would hope to outperform the benchmark, resulting in an excess return greater than zero. Negative excess return indicates the investor would have been better off investing in a low-cost index product. The higher the excess return, the better.

What Are the Limitations?

Excess return does not take into account the level of risk that was undertaken in order to generate the added value.

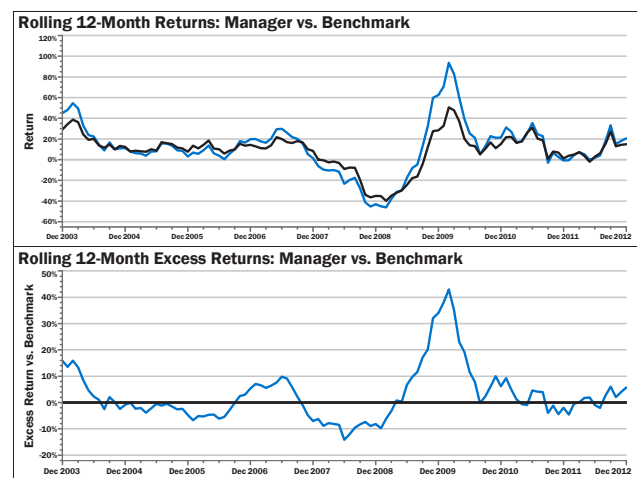
Also, excess return is frequently confused with alpha. The two are not the same. Alpha first adjusts for market-level risk before calculating manager skill. Excess return does not.

Finally, it is important to utilize an appropriate benchmark when analyzing excess return. Failure to use an appropriate benchmark results in an apples-to-oranges comparison.

What Do the Graphs Show Me?

Below we see two different ways of displaying the excess return. The upper graph shows the rolling period returns for both the manager in blue and the benchmark in black. The gap between the two is the excess return.

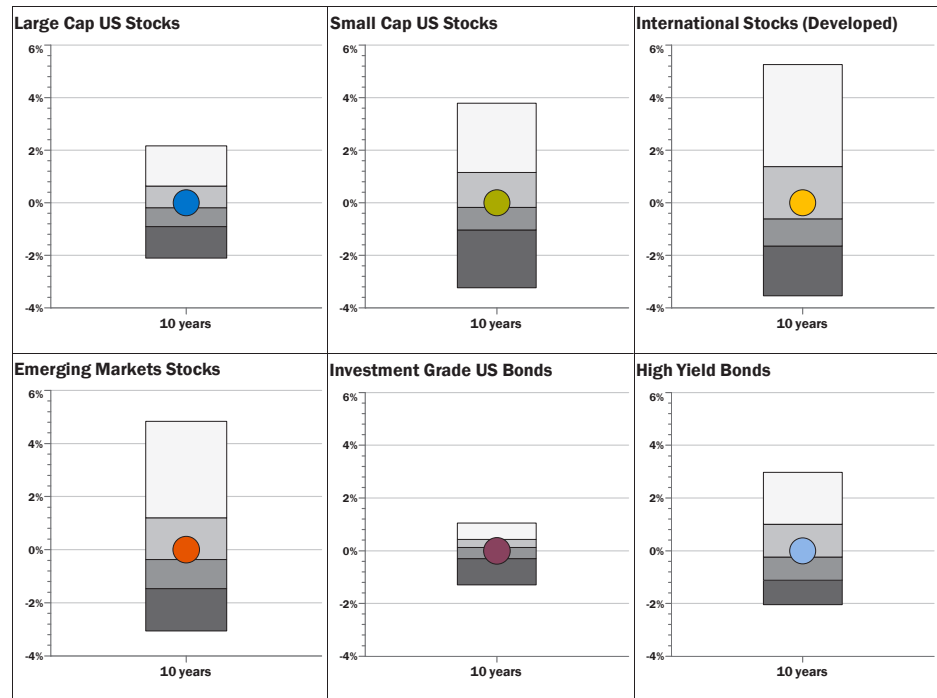
The lower graph is an alternative way of showing the same information. In this case, the flat, horizontal line at zero is the benchmark. The blue line represents the difference between the two, i.e. the excess return.



Excess Return

What Are Typical Values?

The table to the right displays the ranges of 10-year excess returns versus the relevant benchmarks across six asset classes. The universes consist of separately managed account composite returns. According to the data, the median manager typically falls short of the benchmark. Of the four equity asset classes, the dispersion between the best and worst large cap fund is the narrowest, even though the size of the large cap universe is the biggest. This suggests that large cap equity is a fairly efficient asset class.



January 2003 - December 2012 • Symbol = Benchmark Index

Excess Return Funds In the Universe	Large Cap 230	Small Cap 94	International 325	Emerging 64	Gov/Corp 293	HY Bond 96
5th Percentile	2.17%	3.79%	5.27%	4.83%	1.36%	2.97%
25th Percentile	0.64%	1.15%	1.38%	1.20%	0.47%	1.00%
Median	-0.18%	-0.17%	-0.60%	-0.37%	0.09%	-0.24%
75th Percentile	-0.90%	-1.03%	-1.64%	-1.46%	-0.39%	-1.12%
95th Percentile	-2.10%	-3.23%	-3.54%	-3.06%	-1.76%	-2.05%

Related Metrics

Alpha: a measure of “manager skill,” adjusted for the level of market risk

Information Ratio: a manager’s added value and consistency of added value

Math Corner

Excess return is one of the simplest metrics to calculate. Its simplicity is both an advantage and a disadvantage. It is easy to understand, but does not take into consideration any form of risk.

$$\text{Excess Return} = (\text{AnnRtn}(r_1, \dots, r_n) - \text{AnnRtn}(BM_1, \dots, BM_n))$$

Information Ratio

A benchmark-relative, return-versus-risk metric, the Information ratio measures the excess return against the benchmark divided by tracking error, where tracking error is a measure of consistency.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				
TRADE-OFF		INFORMATION RATIO		

How Is it Useful?

The Information ratio answers the two most important questions for an active manager. First, did the manager outperform the passive benchmark? Second, was the manager able to outperform the benchmark consistently? If the answer to either of these is “no,” then a low-cost passive product like an index fund or an ETF might make sense. Therefore, the Information ratio stands as a great way to justify an active manager’s existence.

What Is a Good Number?

The higher the Information ratio, the better. If the Information ratio is less than zero, it means the active manager failed on the first objective of outperforming the benchmark. Of all the performance statistics, the Information ratio is one of the most difficult hurdles to clear. Generally speaking, an Information ratio in the 0.40-0.60 range is considered quite good. Information ratios of 1.00 for long periods of time are rare. Typical values for Information ratios vary by asset class. Details are provided on the reverse side.

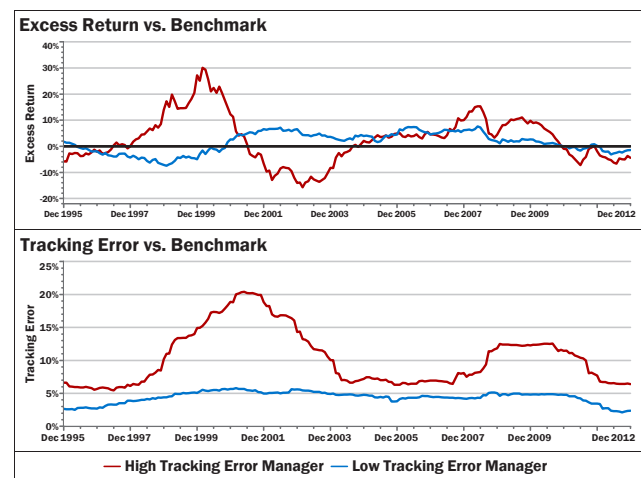
What Are the Limitations?

The Information ratio is a benchmark-relative statistic. It is entirely possible for a manager to have a high Information ratio, but still exhibit significant losses if the benchmark is down.

What Do the Graphs Show Me?

The top graph displays the numerator, the excess return over the benchmark. The thick black line is the benchmark, and the red and blue lines show the rolling excess returns for two different managers. The bottom graph shows the denominator, which is the tracking error versus the benchmark. The smaller the tracking error, the more consistent the excess returns.

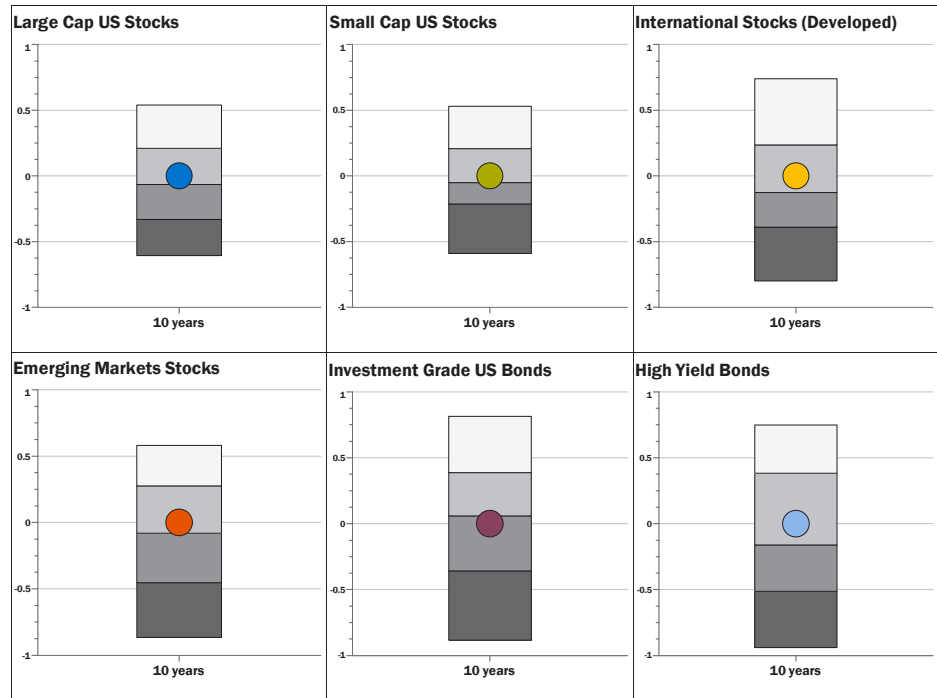
With the red manager, we see that the excess return is higher overall than the blue manager. However, we see the red manager’s excess return pattern is much more erratic, resulting in a higher tracking error. In contrast, the blue manager’s excess returns are lower, but much more consistent. Therefore, the blue manager has a higher Information ratio than the red manager.



Information Ratio

What Are Typical Values?

To the right are ranges of 10-year Information ratios across six asset classes. Peer groups of separately managed account composites are compared to their relevant benchmarks. The difficulty of achieving a high Information ratio stands out. The median manager typically has an Information ratio near or below zero. It is quite rare to see managers with Information ratios in excess of 1.00 over long time periods.



January 2003 - December 2012 • Symbol = Benchmark Index

Information Ratio Funds in the Universe	Large Cap 230	Small Cap 94	International 325	Emerging 64	Gov/Corp 293	HY Bond 96
5th Percentile	0.54	0.53	0.74	0.58	0.81	0.75
25th Percentile	0.21	0.21	0.24	0.28	0.39	0.38
Median	-0.07	-0.05	-0.13	-0.08	0.06	-0.16
75th Percentile	-0.33	-0.21	-0.39	-0.45	-0.36	-0.51
95th Percentile	-0.61	-0.59	-0.80	-0.87	-0.88	-0.94

Related Metrics

Excess Return: the difference between a manager's returns and the benchmark's returns

Tracking Error: the standard deviation of excess returns of a manager versus its benchmark

Alpha: a measure of "manager skill," adjusted for the level of market risk

Math Corner

The numerator of the Information ratio is quite easy to calculate. It is simply the difference between the manager return and its benchmark return. The denominator is calculated by taking the standard deviation of the numerator. It is the volatility of that excess return series. The standard deviation of excess return is known as tracking error.

$$\text{Info Ratio} = \frac{(\text{AnnRtn}(r_1, \dots, r_n) - \text{AnnRtn}(BM_1, \dots, BM_n))}{\text{AnnStdDev}(\text{exc. rtn}_1, \dots, \text{exc. rtn}_n)}$$

Kurtosis

Kurtosis identifies where the volatility risk came from in a distribution of returns. Kurtosis improves one's understanding of volatility risk.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				KURTOSIS
TRADE-OFF				

How Is it Useful?

Kurtosis tells us where the risk exists. On a month-to-month basis, does the investment typically display a moderate amount of risk? Or does the investment appear to have little risk until the risk suddenly hits all at once? Kurtosis tells us whether the risk is spread evenly through the distribution of returns or if it tends to be concentrated in tail events.

What Is a Good Number?

Generally speaking, one would hope to see a low or negative kurtosis. A low or negative kurtosis means that on a period-by-period basis most observations fall within a predictable band. The risk that does occur happens within a moderate range, and there is little risk in the tails. Alternatively, the higher the kurtosis, the more it indicates that the overall risk of an investment is driven by a few extreme "surprises" in the tails of the distribution.

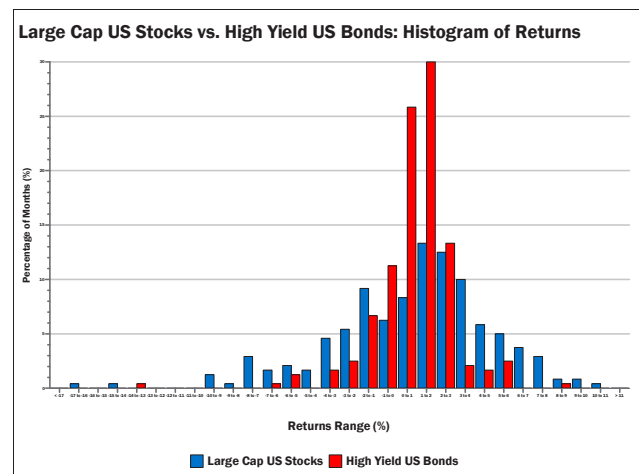
What Are the Limitations?

Kurtosis as a stand-alone metric is not very useful. Kurtosis is only useful when used in conjunction with standard deviation. It is possible that an investment might have a high kurtosis (bad), but the overall standard deviation is low (good). Conversely, one might see an investment with a low kurtosis (good), but the overall standard deviation is high (bad). Kurtosis gives a better understanding of standard deviation, but used in isolation, kurtosis is meaningless.

What Does the Graph Show Me?

The distribution for US Large Cap Stocks is in blue, while High Yield Bonds are in red. The shapes of the two distributions are quite different. Large cap stocks exhibit higher overall risk, as evidenced by the wider range of the distribution. This distribution resembles the classic, bell-shaped curve. While there is more risk overall, the risk is not concentrated in any section of the curve.

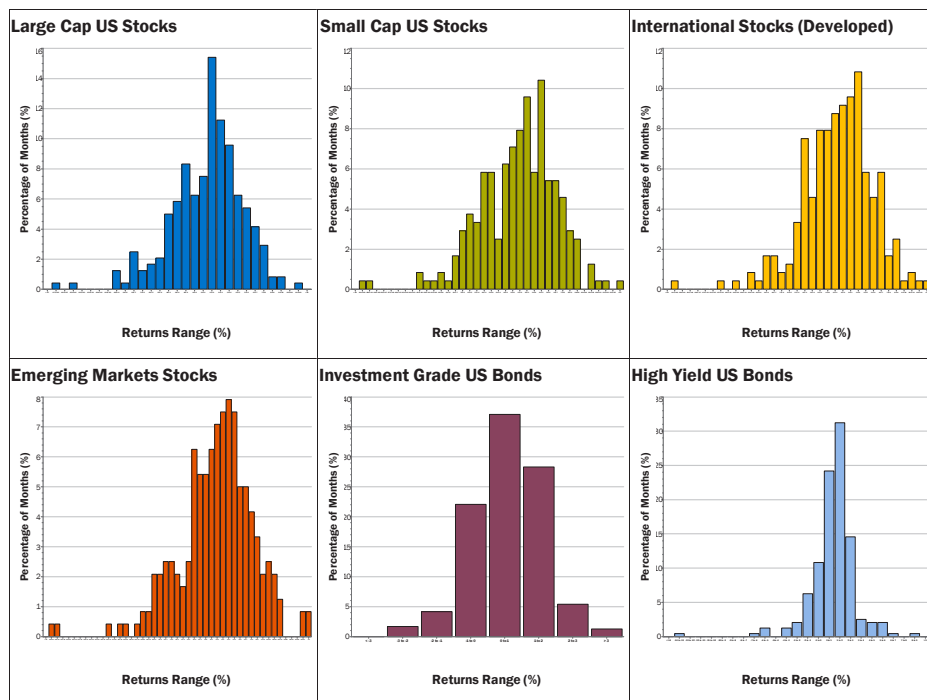
The distribution for High Yield Bonds is quite different. Most of the returns are clustered towards the center. However, on rare occasions, High Yield Bonds exhibit extreme gains or losses, and this is what is meant by high kurtosis. It is the statistical equivalent of the old saying, "When it rains, it pours."



Kurtosis

What Are Typical Values?

A normal distribution would have a kurtosis of 0.0. Investment grade fixed income has kurtosis values closest to 0.0, meaning the return distributions are closest to fitting a normal, bell-shaped curve. High yield bonds have higher kurtosis values, even though their standard deviations tend to be lower than the equity asset classes. The interpretation for high yield bonds is that while the overall risk is lower, when the risk does happen, it tends to be extreme.



January 1986 - December 2012

Skewness (and Kurtosis)	Return	Standard Deviation	Skewness	Kurtosis
Large Cap US Stocks	11.03%	14.73%	-0.82	2.35
Small Cap US Stocks	13.66%	18.96%	-0.81	2.49
International Stocks (Developed)	11.29%	16.66%	-0.80	2.05
Emerging Markets Stocks	18.03%	22.04%	-0.86	2.33
Investment Grade US Bonds	7.54%	3.77%	-0.14	0.59
High Yield US Bonds	10.03%	7.06%	-1.31	8.11

Related Metrics

Standard Deviation: the degree to which individual returns diverge from the average return

Skewness: a measure of the direction and degree that a set of returns is tilted by its outliers

Omega: the count and scale of returns above versus below a breakpoint

Math Corner

Kurtosis is also known as the fourth moment of the distribution, used in conjunction with mean, standard deviation, and skewness to understand the shape of a distribution of returns. In its base case, kurtosis has a neutral value of 3.0. The calculation is frequently modified by the second term in the equation below, which scales kurtosis so that the baseline, neutral value is 0.0.

$$Kurtosis(r_1, \dots, r_n) = \frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum_{i=1}^n \left(\frac{r_i - \bar{r}}{\sigma} \right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)}$$

Maximum Drawdown

A risk metric indicating capital preservation, the maximum drawdown measures the peak-to-trough loss of an investment.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK			MAXIMUM DRAWDOWN	
TRADE-OFF				

How Is it Useful?

Maximum drawdown offers investors a worst case scenario. Maximum drawdown tells the investor how much would have been lost if an investor bought at the absolute peak value of an investment, rode it all the way down, and sold at rock-bottom.

What Is a Good Number?

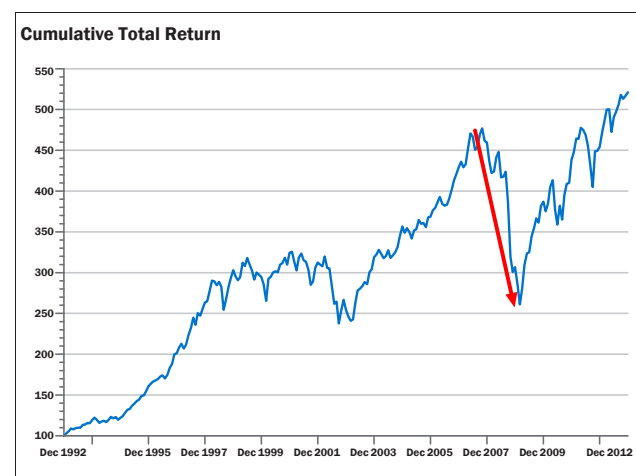
One would hope that the maximum drawdown would be as small as possible. If an investment never lost a penny, the maximum drawdown would be zero. The worst possible maximum drawdown would be 100%, meaning the investment is completely worthless. Most maximum drawdowns will fall somewhere between these two extremes. The two most important elements to keep in mind when analyzing maximum drawdown are the asset class and time frame being analyzed.

What Are the Limitations?

Maximum drawdown only measures one dimension of capital preservation. It does not tell us how long it took to recover from the loss, or if the investment even recovered at all. Maximum drawdown does not tell us if other losses were short and sharp or long and drawn-out. Maximum drawdown measures only the largest drawdown. Other smaller periods of loss are ignored. For these reasons, Zephyr recommends the pain index as a more complete measure of the depth, duration, and frequency of losses.

What Does the Graph Show Me?

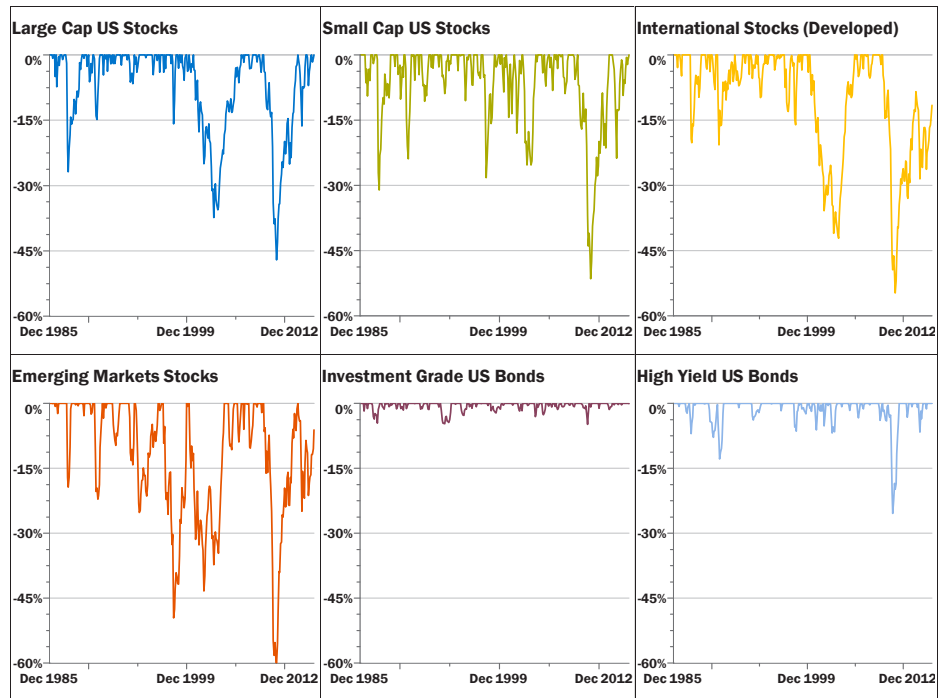
The below graph is a simple cumulative return chart showing how the value of \$100 would have fluctuated over time. The red arrow represents the maximum drawdown. It shows, in percentage terms, how much wealth was lost from peak-to-trough of the worst decline. Note that smaller peak-to-trough losses are not quantified.



Maximum Drawdown

What Are Typical Values?

The maximum drawdown calculation depends heavily upon the time frame under examination. In this example, we examine the period from January 1986 to December 2012. For many asset classes, the worst period fell between late 2007 and early 2009, during the Credit Crisis. Many equity asset classes lost over half of their value. It took over two years for most equity asset classes to recover, but as of December 2012, International Stocks have yet to fully recoup their losses.



January 1986 - December 2012

	Maximum Drawdown	Maximum Drawdown Begin Date	Maximum Drawdown End Date	Maximum Drawdown Length	Maximum Drawdown Recovery Date
Large Cap US Stocks	-47.04%	Nov-07	Feb-09	16	Apr-11
Small Cap US Stocks	-51.49%	Nov-07	Feb-09	16	Dec-10
International Stocks (Developed)	-54.75%	Nov-07	Feb-09	16	N/A
Emerging Markets Stocks	-60.38%	Nov-07	Feb-09	16	Apr-11
Investment Grade US Bonds	-4.84%	Mar-08	Oct-08	8	Dec-08
High Yield US Bonds	-25.43%	Nov-07	Nov-08	13	Jul-09

Related Metrics

Pain Index: the depth, duration, and frequency of losses

Down Capture: the percentage of the benchmark's negative returns experienced by the manager

Downside Deviation: the amount of "bad" volatility risk

Math Corner

The calculation of maximum drawdown looks at all subperiods of the time period in question and calculates the compound return of the manager over each subperiod. The maximum drawdown is the lowest value of all these compound returns.

$$MaxDD(T) = Max_{\tau \in (0, T)} [Max_{t \in (0, \tau)} r(t) - r(\tau)]$$

Omega

Omega compares upside gains against downside risks. Omega represents the count and scale of returns above a breakpoint versus the count and scale of observations below a breakpoint.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				
TRADE-OFF				OMEGA

How Is it Useful?

Omega represents one useful way of understanding tail risk, the impact that extreme observations have on an overall set of numbers. If the returns of a manager are close to the minimum acceptable return (MAR) breakpoint, they don't strongly affect omega. However, if many returns lie above or below the MAR, or if the returns are extreme, those returns will impact the value of omega significantly.

What Is a Good Number?

Similar to all of the other return-versus-risk ratios, the higher the omega, the better. One hopes to see many returns above the MAR. Conversely, one hopes to see few observations below the MAR.

Omega is an absolute-return way of quantifying return and risk, so there is no breakpoint above which an omega can be considered good. Omega for an individual manager must be compared to omega for an index or omegas for a peer group to be placed in a proper context.

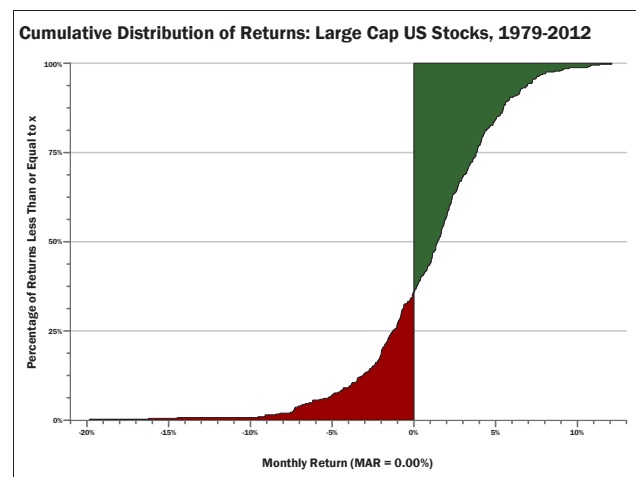
What Are the Limitations?

Omega compares the good observations against the bad observations, rolled into a single number. One might want to separate these two elements and observe them independently. Two other statistics, upside omega and downside omega, accomplish this.

What Does the Graph Show Me?

The graph below orders the monthly returns from worst-to-first. On the left-most portion of the graph is the worst month, and from left to right are the second-worst months, third-worst months, etc. until we get to the best, highest returns on the far right of the S-curve. The graph also shows the range in which most of the observations occur.

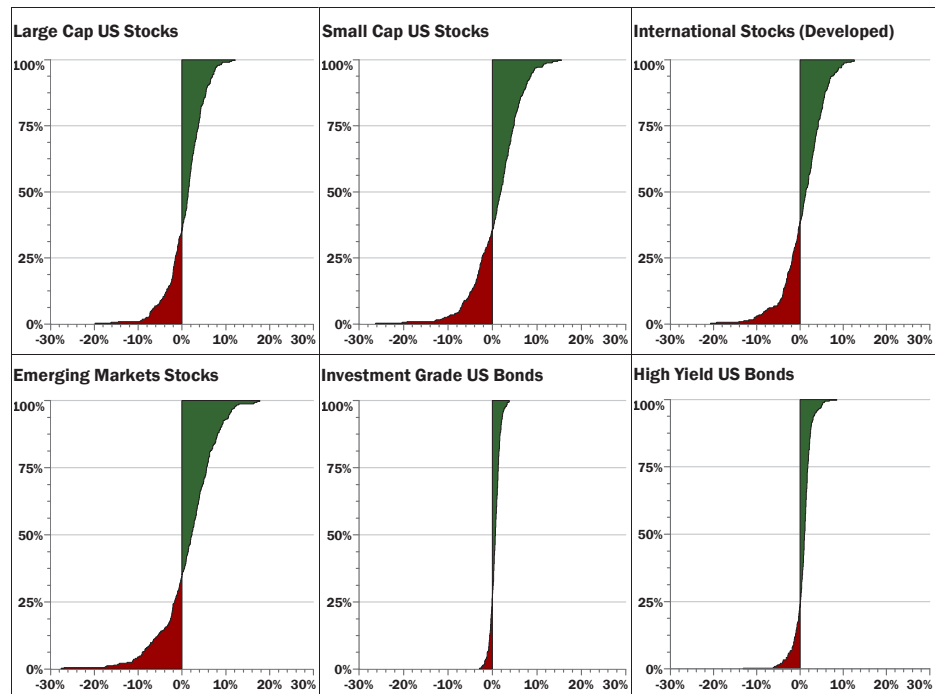
The graph is split by the MAR, in this case, zero percent. The count and scale of observations above the MAR line are shaded in green. This area should be large. The count and scale of observations below the MAR are shaded in red. This area should be small. Omega is the green area divided by the red area.



Omega

What Are Typical Values?

One would assume that omega would have very different values during the bull markets of the 1980s and 1990s versus the bear market-dominated 2000s. However, looking at the raw numbers, it isn't immediately obvious there is a big difference between decades. One must look more closely and think back to what omega is showing us to recognize the sizable difference between decades. Keep in mind omega represents the good, green area divided by the bad, red area in the graphs to the right. During the 1980s and 1990s, the ratio of green-to-red typically stood at 2-to-1. During the difficult 2000s, the ratios for many asset classes were closer to 1.25-to-1, meaning there was almost as much bad, red area as good, green area.



January 1986 - December 2012

Omega	1980s	1990s	2000s	Common 1/86 - 12/12
Large Cap US Stocks	2.79	2.74	1.15	1.80
Small Cap US Stocks	2.36	2.26	1.36	1.77
International Stocks (Developed)	2.87	1.97	1.26	1.72
Emerging Markets Stocks	N/A	1.75	1.56	1.92
Investment Grade US Bonds	3.33	4.37	3.54	4.14
High Yield US Bonds	3.18	4.50	2.09	2.96

Related Metrics

Upside Omega: the count and scale of returns above a breakpoint

Downside Omega: the count and scale of returns below a breakpoint

Skewness: a measure of the direction and degree that a set of returns is tilted by its outliers

Kurtosis: an indicator of where the volatility of an investment came from

Math Corner

Omega was first proposed by Con Keating and William Shadwick in their 2002 paper "A Universal Performance Measure". Omega is the ratio of two integrals: the area above the minimum acceptable return (MAR) in the numerator and the area below the MAR as the denominator. Omega captures all four moments of the distribution (return, standard deviation, skewness, and kurtosis) in a single measure.

$$Omega = \frac{\sum_{r_i > MAR} (r_i - MAR)}{\sum_{r_i < MAR} (MAR - r_i)}$$

Pain Index

A proprietary risk metric, the pain index quantifies the capital preservation tendencies of a manager or index. It measures the depth, duration, and frequency of periods of losses.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK			PAIN INDEX	
TRADE-OFF				

How Is it Useful?

Losing money is a painful experience. The pain index attempts to measure the complete scope of losses. It addresses the shortcoming of only looking at maximum drawdown. It measures risk in terms of absolute returns.

What Is a Good Number?

There is no hard-and-fast rule or breakpoint that separates a good pain index from a bad one. One must compare a manager's pain index against an appropriate benchmark or peer group in order to gain an understanding of whether a manager's pain index is good or bad.

It can be said that the lower the number, the better. The investor would prefer, 1) smaller overall losses, 2) shorter periods of loss, and 3) infrequent losses. All three of these would translate to a smaller pain index. A pain index of zero would represent the best possible outcome, meaning the investment never lost value.

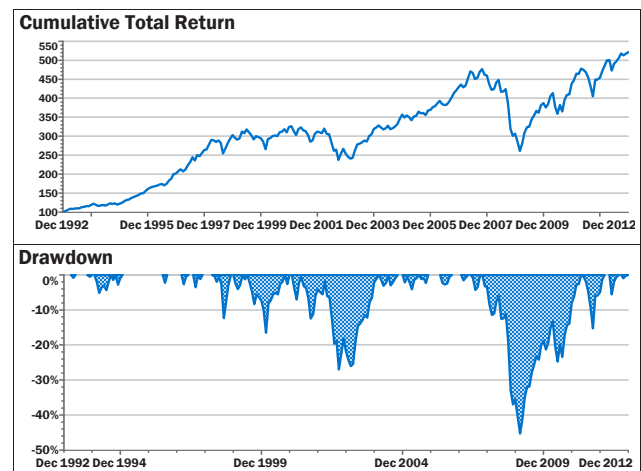
What Are the Limitations?

The pain index completely ignores the upside. An investment with a very high pain index, signifying a lot of losses, might also have a large number of gains too. The gains are not captured by the pain index.

What Do the Graphs Show Me?

The upper half of the image is simply a cumulative return graph, commonly known as a "Growth of \$100" chart. The drawdown graph beneath hones in on the periods of losses. The drawdown graph illustrates what the pain index quantifies. The depth, duration, and frequency of losses are seen on the drawdown graph.

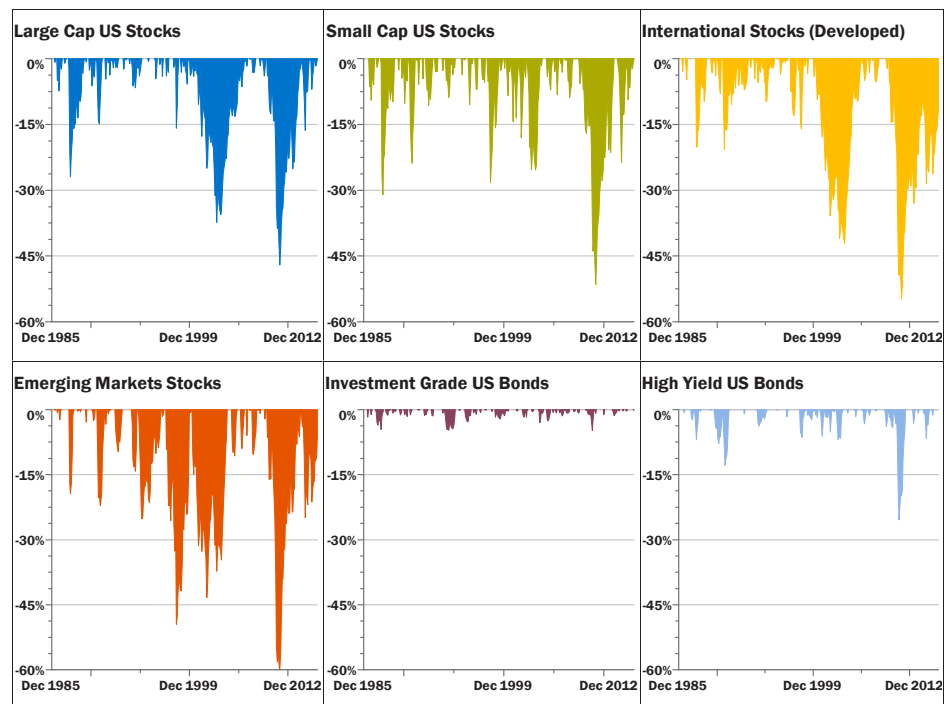
The pain index quantifies the losses illustrated on the drawdown graph. It is the shaded area on the drawdown graph. Obviously, the investor hopes this area to be as small as possible.



Pain Index

What Are Typical Values?

Unfortunately, there isn't a single value that can be described as "typical" when it comes to understanding the pain index. There are two very important elements one should keep in mind, namely, the asset class under consideration and the time frame. Risky asset classes, like emerging market equities, have exhibited periods of significant losses. Investment grade bonds, on the other hand, have historically experienced very little "pain" area. The other key contextual element is the time frame. In the 1980s and 1990s, losses were few and far between. In the 2000s, losses were much more common with the Dot-Com Bust and the Credit Crunch.



ASSET CLASS	1980's	1990's	2000's	Common 1/88-12/10
Large Cap Stocks (US)	4.67%	1.80%	19.07%	10.11%
Small Cap Stocks (US)	7.83%	5.05%	13.58%	9.16%
Int'l Developed	4.04%	7.76%	19.33%	13.29%
Emerging Markets	N/A	14.03%	18.90%	16.06%
Invst Grade Bonds (US)	1.57%	0.84%	0.48%	0.63%
High Yield Bonds (US)	N/A	1.47%	3.48%	2.37%
REITs	2.71%	5.26%	11.32%	9.74%
Commodities	8.30%	16.60%	16.83%	17.39%
Hedge Funds	2.74%	0.51%	2.95%	2.06%

Related Metrics

Pain Ratio: the trade-off of return per unit of pain index risk

Down Capture: the percentage of the benchmark's negative returns experienced by the manager

Downside Deviation: the amount of "bad" volatility risk

Maximum Drawdown: the largest peak-to-trough losses

Math Corner

The Pain Index is derived by calculating an integral, measuring the area between a curve (e.g., the drawdowns) and the zero line separating periods of gains from losses. The integral is then divided by the length of the X-axis, representing the time frame.

$$\text{Pain Index} = \text{abs} \left(\frac{\int_{t_1}^{t_2} D(x) dx}{(t_2 - t_1)} \right)$$

Pain Ratio

A proprietary return-versus-risk trade-off metric, the pain ratio compares the added value over the risk-free rate against the depth, duration and frequency of losses.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				
TRADE-OFF			PAIN RATIO	

How Is it Useful?

While one certainly wants to minimize losses, it is also important to make money. The pain ratio quantifies this trade-off into a single number. The pain ratio compares the gains over the risk-free investment against the losses that were suffered to obtain that return.

What Is a Good Number?

The pain ratio is similar to the Sharpe ratio, but with a different definition of risk. The Sharpe ratio measures volatility risk as standard deviation. The pain ratio uses the pain index as the key element in measuring capital preservation risk.

With both ratios, the higher the number, the better. However, there is no hard-and-fast breakpoint above which one can say the ratios are good or bad. One must compare the pain ratio of a manager to the pain ratios of peers or an index to ascertain whether it is relatively better or worse than the alternatives.

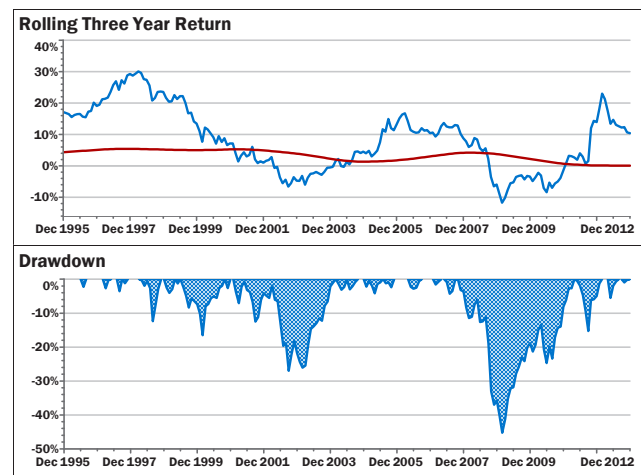
What Are the Limitations?

Like the Sharpe ratio, the pain ratio can exhibit negative values. If the time period analyzed represents a bear market when the investment underperformed the risk-free rate, the pain ratio will be negative. Therefore, one shouldn't set hard-target screens that eliminate managers with a negative pain ratio.

What Do the Graphs Show Me?

The upper graph shows the return metric, the added value above and beyond the risk-free rate, in red. One hopes to outperform the risk-free rate, and by a large margin. However, there will be times when risky investments fall short of the risk-free rate.

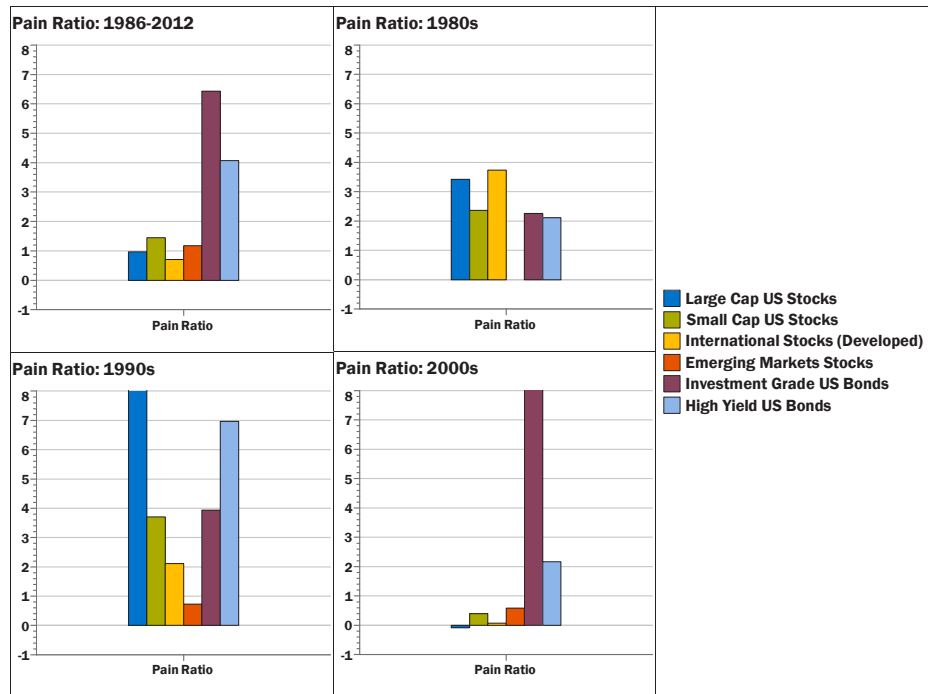
The lower graph shows the risk metric of the pain ratio. The risk quantified by the pain ratio is capital preservation risk. The metric used in the denominator is the pain index. The pain index represents the depth, duration, and frequency of losses and is measured in the area seen below. One would hope that this area would be as small as possible. These two graphs depict the pain ratio's numerator and denominator, respectively.



Pain Ratio

What Are Typical Values?

There is no single value that could be described as “typical” when understanding the pain ratio. The values for the pain ratio will be heavily influenced by the asset class and the time frame under consideration. With the pain ratio, both the numerator and denominator will be impacted by the general market environment. In the 1980s and 1990s, excess returns were high and losses were low, leading to high (i.e. good) pain ratios. In the 2000s, excess returns were lower and losses more frequent, so pain ratios were much lower.



January 1986 - December 2012

Pain Ratio	1980s	1990s	2000s	Common 1/86 - 12/12
Large Cap US Stocks	3.42	8.11	-0.09	0.96
Small Cap US Stocks	2.36	3.70	0.40	1.45
International Stocks (Developed)	3.74	2.11	0.07	0.71
Emerging Markets Stocks	N/A	0.73	0.59	1.17
Investment Grade US Bonds	2.26	3.94	8.11	6.44
High Yield US Bonds	2.11	6.97	2.17	4.07

Related Metrics

Pain Index: the depth, duration, and frequency of losses

Sharpe Ratio: the trade-off of return per unit of volatility risk

Sortino Ratio: the trade-off of return per unit of downside volatility risk

Math Corner

The return element of the pain ratio is the annualized return of the investment in excess of the risk-free investment. Typically, a short-term cash investment is used as the risk-free investment. The denominator of the pain ratio is the pain index, an integral measuring the depth, duration, and frequency of losses.

$$\text{Pain Ratio} = \frac{(\text{AnnRtn}(r_1, \dots, r_n) - \text{AnnRtn}(c_1, \dots, c_n))}{\text{Pain Index}(r_1, \dots, r_n)}$$

R-Squared

R-squared represents the “goodness of fit” of a manager to its benchmark. R-squared is the percentage of variation in a manager’s returns explained by the benchmark’s returns.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK		R-SQUARED		
TRADE-OFF				

How Is it Useful?

R-squared is used primarily as a cross-check on the appropriateness of the benchmark. Many other statistics such as alpha, beta, Information ratio, and up/down capture use a passive benchmark as a reference point. If the R-squared of a manager to the benchmark is too low, the usefulness of all these other benchmark-relative metrics diminishes.

What Is a Good Number?

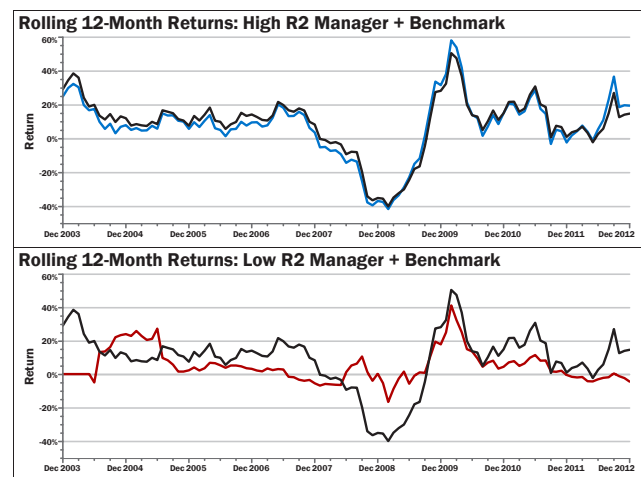
Generally speaking, in “efficient” asset classes, like large cap stocks and investment grade bonds, analysts look for higher R-squared numbers in the +85% range. For “inefficient” asset classes, like small cap, foreign, or emerging market stocks, investors would be more liberal and accept lower R-squared numbers in the +70% range. The assumption with inefficient asset classes is that managers should be more active and not track the benchmark too closely. Suffice it to say, if the R-squared is 50% or less, metrics like alpha, beta, or up/down capture will be of limited use.

What Are the Limitations?

R-squared is of limited value as a stand-alone metric. It doesn’t measure outperformance or underperformance. It just describes how closely the manager tracked the benchmark. R-squared is best used as a preliminary cross-check to the benchmark’s appropriateness.

What Do the Graphs Show Me?

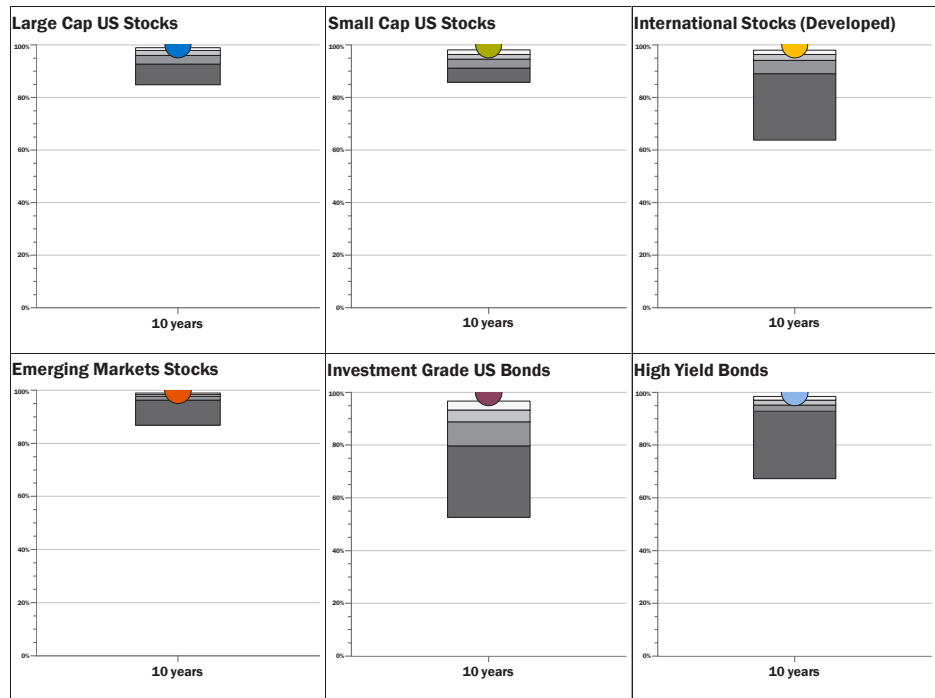
In the two graphs below, the black line is identical. It is a rolling, three-year return for the benchmark. Superimposed over the benchmark is a high- R-squared manager in blue (upper) and a low- R-squared manager in red (lower). As the benchmark black line zigs and zags, the blue manager is moving almost in lockstep with it. The majority of the movement in the manager can thus be explained or attributed to the movements in the benchmark. In contrast, the red manager doesn’t seem to track the benchmark very well at all. The movement of the red manager seems to be independent of the benchmark. This results in a low R-squared. Therefore, it wouldn’t be very useful to rely on other benchmark-relative metrics (e.g. alpha) when looking at the red manager.



R-Squared

What Are Typical Values?

On the right are typical R-squared ranges for six peer groups over 10 years. Within traditional asset classes, most managers can attribute the majority of the variation in performance to movements in the broad market. The good news for those managers is that the other performance-based metrics, like alpha, Information ratio, up/down capture, etc. will be relevant and useful. For the managers in the bottom quartile of R-squared, it might be more useful to look for a more appropriate benchmark or to examine non-benchmark relative metrics, like the Sharpe ratio and pain ratio, for performance analysis.



January 2003 - December 2012 • Symbol = Benchmark Index

R-Squared Funds In the Universe	Large Cap 230	Small Cap 94	International 325	Emerging 64	Gov/Corp 293	HY Bond 96
5th Percentile	99.03%	98.22%	98.02%	98.90%	96.72%	98.46%
25th Percentile	97.97%	96.36%	96.43%	98.35%	93.33%	97.05%
Median	96.00%	94.69%	94.16%	97.62%	88.82%	95.19%
75th Percentile	92.78%	91.24%	89.08%	96.25%	79.70%	92.79%
95th Percentile	84.86%	85.77%	63.86%	86.87%	52.61%	67.26%

Related Metrics

Beta: the sensitivity of a manager to a benchmark

Tracking Error: the standard deviation of excess returns of a manager versus its benchmark

Math Corner

R-squared literally takes the correlation of a manager versus a benchmark and squares it. Squaring correlation removes the directional aspect of correlation. It is impossible to have a negative R-squared. This is intentional, as the sole point of R-squared is to determine what percentage, from 0% to 100%, of the variation of a manager's return is explained by the benchmark.

$$R - Squared = \left(\frac{\sum_{i=1}^n (r_i - \bar{r}) * (BM_i - \overline{BM})}{\sqrt{\sum_{i=1}^n (r_i - \bar{r})^2} * \sqrt{\sum_{i=1}^n (BM_i - \overline{BM})^2}} \right)^2$$

Sharpe Ratio

The most famous return-versus-risk measurement, the Sharpe ratio, represents the added value over the risk-free rate per unit of volatility risk.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				
TRADE-OFF	SHARPE RATIO			

How Is it Useful?

The Sharpe ratio simplifies the options facing the investor by separating investments into one of two choices, the risk-free rate or anything else. Thus, the Sharpe ratio allows investors to compare very different investments by the same criteria. Anything that isn't the risk-free investment can be compared against any other investment. The Sharpe ratio allows for apples-to-oranges comparisons.

What Is a Good Number?

Sharpe ratios should be high, with the larger the number, the better. This would imply significant outperformance versus the risk-free rate and/or a low standard deviation. However, there is no set-in-stone breakpoint above, which is good, and below, which is bad. The Sharpe ratio should be compared against an index or an appropriate peer group. Keep in mind that it is possible for Sharpe ratios to be negative. If the investment has negative returns and falls short of the risk-free rate, the Sharpe ratio will be negative.

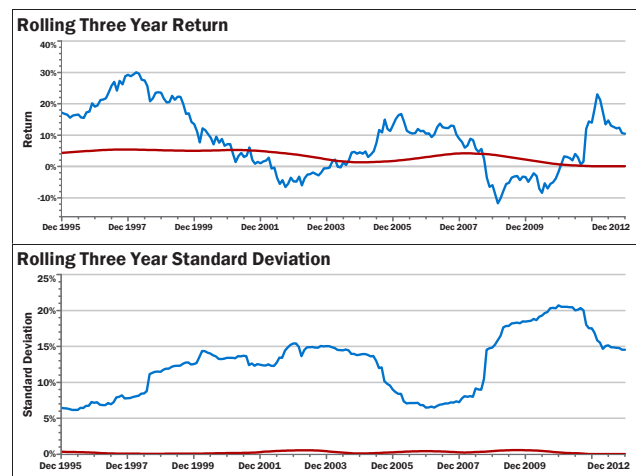
What Are the Limitations?

The Sharpe ratio defines risk as standard deviation, so it carries the limitations inherent in standard deviation. Standard deviation fails to differentiate between upside deviation and downside deviation. Also, standard deviation does not take into account the timing of returns.

What Do the Graphs Show Me?

The graphs below illustrate the two halves of the Sharpe ratio. The upper graph shows the numerator, the excess return over the risk-free rate. The blue line is the investment. The red line is the risk-free rate on a rolling, three-year basis. More often than not, the investment's return exceeds that of the risk-free rate, leading to a positive numerator.

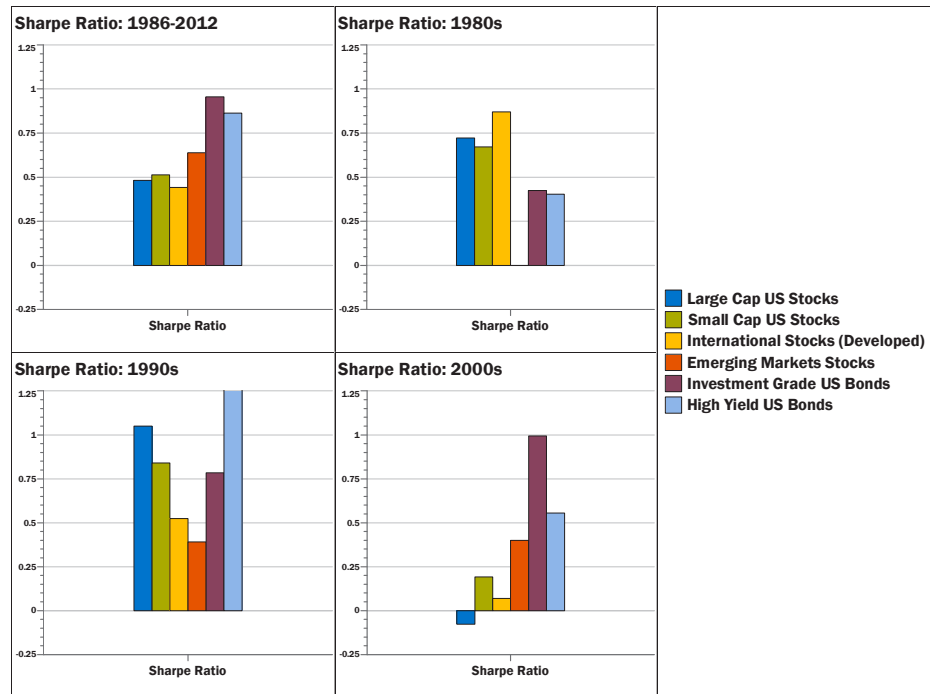
The lower graph shows the risk metric used in the denominator, standard deviation. Standard deviation measures how volatile an investment's returns have been.



Sharpe Ratio

What Are Typical Values?

Historically speaking, fixed income investments have offered the best return-versus-risk trade-offs, when defined by the Sharpe ratio. International stocks have fared the worst, once volatility is weighed against return. Sharpe ratios were lower in the 2000s compared to the 1980s and 1990s. Featuring two large bear markets, many equity asset classes struggled, or even failed to outperform, cash during this decade. Some Sharpe ratios were negative in the 2000s. Some Sharpe ratios were negative in the 2000s.



January 1986 - December 2012

Sharpe Ratio	1980s	1990s	2000s	Common 1/86 - 12/12
Large Cap US Stocks	0.72	1.05	-0.08	0.48
Small Cap US Stocks	0.67	0.84	0.19	0.51
International Stocks (Developed)	0.87	0.52	0.07	0.44
Emerging Markets Stocks	N/A	0.39	0.40	0.64
Investment Grade US Bonds	0.42	0.78	0.99	0.96
High Yield US Bonds	0.40	1.26	0.56	0.86

Related Metrics

Standard Deviation: the degree to which individual returns diverge from the average return

Zephyr K-Ratio: the degree and consistency of wealth creation

Sortino Ratio: the trade-off of return per unit of downside volatility risk

Math Corner

First proposed by William Sharpe in his landmark 1966 paper "Mutual Fund Performance," the original version of the Sharpe ratio was known as the reward-to-variability ratio. Sharpe revised the formula in 1994 to acknowledge that the risk-free rate used as the reference point is variable, not a constant.

$$\text{Sharpe Ratio} = \frac{(\text{AnnRtn}(r_1, \dots, r_n) - \text{AnnRtn}(c_1, \dots, c_n))}{\text{AnnStdDev}(r_1, \dots, r_n)}$$

Skewness

Skewness measures to what direction and degree a set of returns is tilted, or “skewed,” by its extreme outlier occurrences.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				SKEWNESS
RISK				
TRADE-OFF				

How Is it Useful?

One way of thinking about skewness is that it compares the length of the two “tails” of the distribution. Another way of thinking of skewness is that it measures whether or not the distribution of returns is symmetrical around the mean. The two are related, because if the distribution is impacted more by negative outliers than positive outliers (or vice versa), the distribution will no longer be symmetrical. Therefore, skewness tells us how outlier events impact the shape of the distribution.

What Is a Good Number?

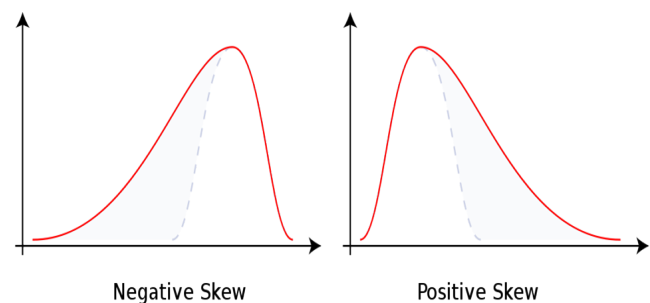
Generally speaking, one would prefer positive skewness. However, in the real world, few investments exhibit a positive skew. Therefore, one might seek investments with skew that is “less negative” than the alternatives.

What Are the Limitations?

Skewness provides valuable information about the distribution of returns. However, skewness must be viewed in conjunction with the overall level of returns. Skewness by itself isn’t very useful. It is entirely possible to have positive skewness (good), but an average annualized return with a low or negative value (bad).

What Does the Graph Show Me?

The below graphs illustrate the difference between a negatively skewed distribution on the left and a positively skewed distribution to the right. The distribution formed by the red line and the shaded grey line in both graphs form a symmetrical distribution. The count and scale of observations above the mean is perfectly balanced by the count and scale of observations below the mean, so the left and right sides of the bell curve are mirror images. However, if one side of the distribution is dominated by its outliers, the distribution is said to be skewed. The left graph illustrates a case where the length of the negative tail is dominant, leading to a negative skew. The graph on the right is the opposite case and represents a positive skew.

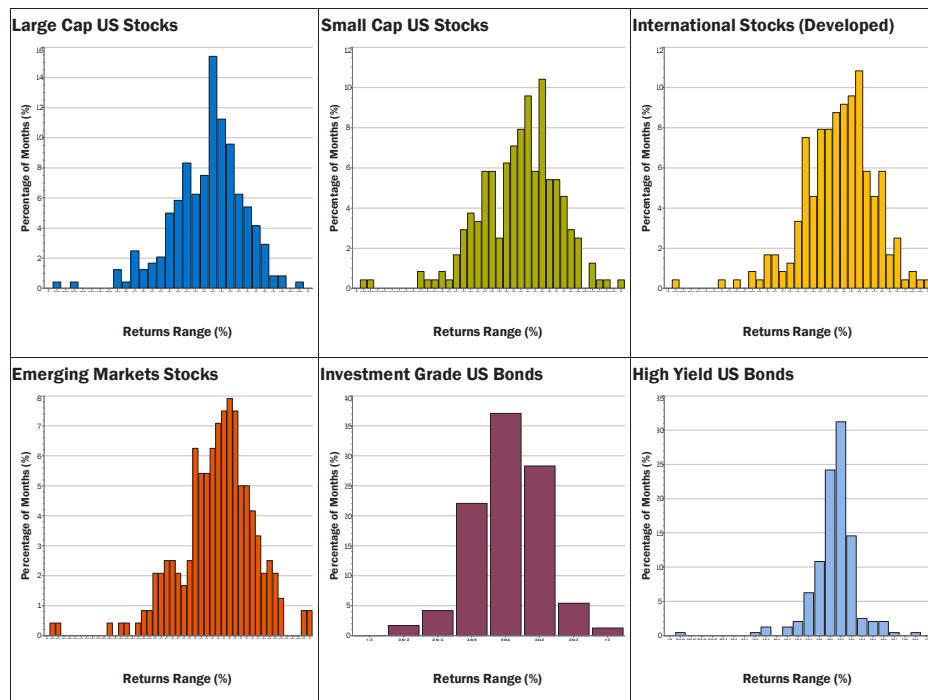


Source: Wikipedia

Skewness

What Are Typical Values?

Positive skewness is preferred, but uncommon. Looking across various asset classes and time periods, one notices the prevalence of negative numbers. Knowing how markets behave, this makes sense. When markets melt down, they tend to melt down in a dramatic fashion. Think of the Credit Crisis, the Dot-Com Bust, the Asian Contagion, or the Long Term Capital Management Crisis. On the upside, gains tend to be less dramatic. While the overall, long-term returns of the markets are positive, those gains come in slower, steadier gains than big bursts. The worst of the worst months tend to be more extreme than the best of the best months. This is what is meant by negative skewness.



January 1986 - December 2012

Skewness (and Kurtosis)	Return	Standard Deviation	Skewness	Kurtosis
Large Cap US Stocks	11.03%	14.73%	-0.82	2.35
Small Cap US Stocks	13.66%	18.96%	-0.81	2.49
International Stocks (Developed)	11.29%	16.66%	-0.80	2.05
Emerging Markets Stocks	18.03%	22.04%	-0.86	2.33
Investment Grade US Bonds	7.54%	3.77%	-0.14	0.59
High Yield US Bonds	10.03%	7.06%	-1.31	8.11

Related Metrics

Kurtosis: an indicator of where the volatility of an investment came from

Omega: the count and scale of returns above versus below a breakpoint

Math Corner

Skewness is also known as the third moment of the distribution. By cubing the differences of the individual observations away from the mean, positive or negative values are possible, which indicate the tilt of the distribution. The process of cubing exacerbates the deviations from the mean, which is why skewness is used for measuring tail risk.

$$Skewness (r_1, \dots, r_n) = \frac{n}{(n-1)(n-2)} \sum_{i=1}^n \left(\frac{r_i - \bar{r}}{\sigma} \right)^3$$

Sortino Ratio

A variation of the Sharpe ratio, the Sortino ratio is a return-versus-risk trade-off metric that uses downside deviation as its measure of risk.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				
TRADE-OFF	SORTINO RATIO			

How Is it Useful?

The Sortino ratio addresses a shortcoming of using standard deviation as a measure of risk in a return-versus-risk trade-off ratio. Standard deviation punishes a manager equally for “good” risk and “bad” risk. Downside deviation adjusts for this by only counting the “bad” risk and ignoring “good” observations in a return series. The Sortino ratio replaces standard deviation with downside deviation, so it is the added return per unit of “bad” risk rather than overall risk.

What Is a Good Number?

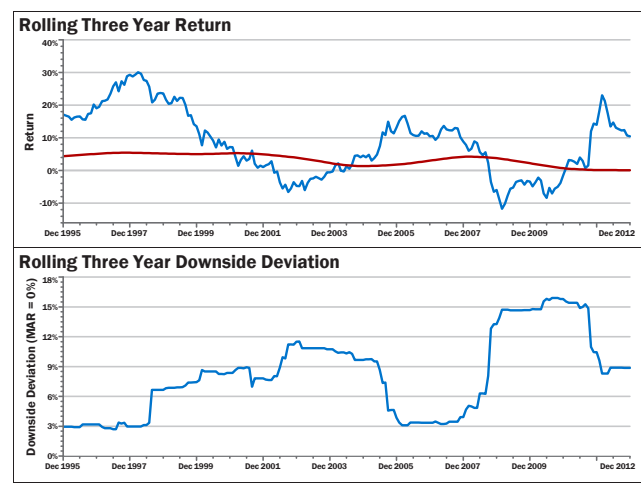
Like most ratios, the higher the Sortino ratio, the better. One would hope to see substantial excess return above and beyond the risk-free rate, accompanied by little downside deviation. A scenario such as this would produce a large Sortino ratio. It is important to keep in mind the asset class under consideration when analyzing Sortino ratios.

What Are the Limitations?

Since the Sortino ratio uses downside deviation as its measure of risk, any limitations of downside deviation carry over to the Sortino ratio. With downside deviation, there must be enough “bad” observations in order for the calculation to be statistically significant.

What Do the Graphs Show Me?

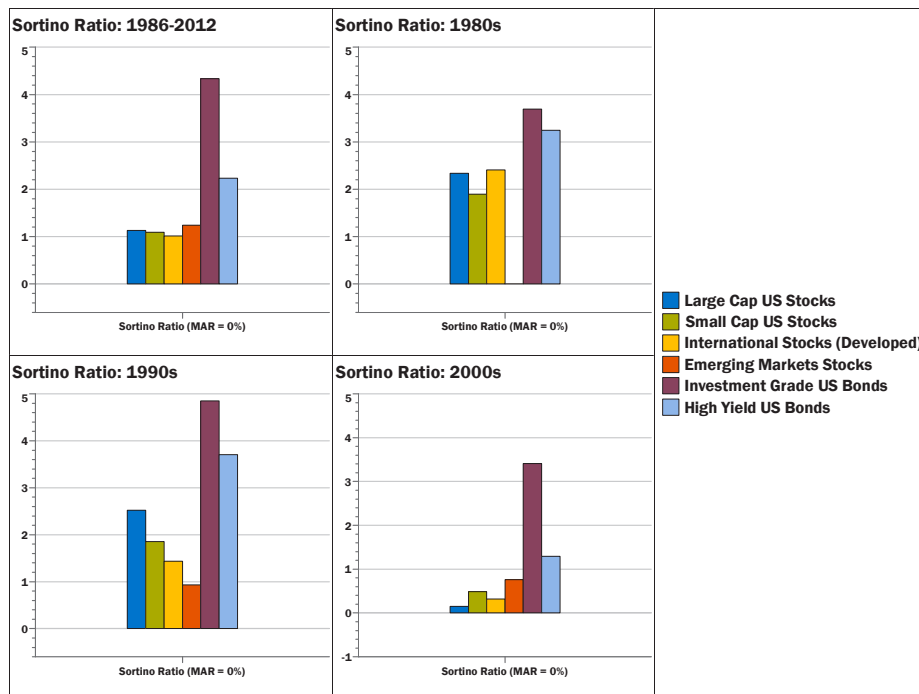
The two graphs below illustrate the two halves of the Sortino ratio. The numerator is identical to the numerator in the Sharpe ratio. It is the rolling excess return above and beyond the risk-free rate, as displayed in the upper graph. The lower graph illustrates how the Sortino ratio uses downside deviations, or the “bad” occurrences in a data stream, as its measure of volatility risk.



Sortino Ratio

What Are Typical Values?

One would expect to see Sortino ratios change significantly for most asset classes between the two decades of the 1980s and 1990s and the “lost decade” of the 2000s. Indeed, that is the case. The numerator of the Sortino ratio was reduced in the 2000s as many asset classes struggled to outperform the risk-free cash rate. The denominator was increased, as markets exhibited more downside deviations short of the 0.0% minimum acceptable return (MAR).



January 1986 - December 2012

Sortino Ratio	1980s	1990s	2000s	Common 1/86 - 12/12
Large Cap US Stocks	2.34	2.52	0.15	1.13
Small Cap US Stocks	1.89	1.86	0.49	1.09
International Stocks (Developed)	2.41	1.44	0.32	1.02
Emerging Markets Stocks	N/A	0.93	0.76	1.24
Investment Grade US Bonds	3.69	4.85	3.41	4.33
High Yield US Bonds	3.25	3.71	1.29	2.23

Related Metrics

Downside Deviation: the amount of “bad” volatility risk

Sharpe Ratio: the trade-off of return per unit of volatility risk

Math Corner

The below calculation for the Sortino ratio is not complicated, as it is simply a variation of the Sharpe ratio. It is up to the user to define what the breakpoint is for minimum acceptable return (MAR) in the calculation of downside risk. Frequently used values for MAR are the risk-free rate or a hard-target value like 0%.

$$\text{Sortino Ratio} = \frac{(\text{AnnRtn}(r_1, \dots, r_n) - \text{AnnRtn}(c_1, \dots, c_n))}{\text{AnnDownDev}(r_1, \dots, r_n)}$$

Standard Deviation

Standard deviation measures how closely returns track their long-term average. Standard deviation measures volatility risk.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK	STANDARD DEVIATION			
TRADE-OFF				

How Is it Useful?

Despite being the oldest way of looking at risk, standard deviation remains applicable. Highly volatile investments are hard for some people to stomach. Also, for those investors who are prone to taking the worst action at the worst time (e.g. chasing returns, or buying high and selling low), highly volatile investments offer many opportunities to make mistakes.

What Is a Good Number?

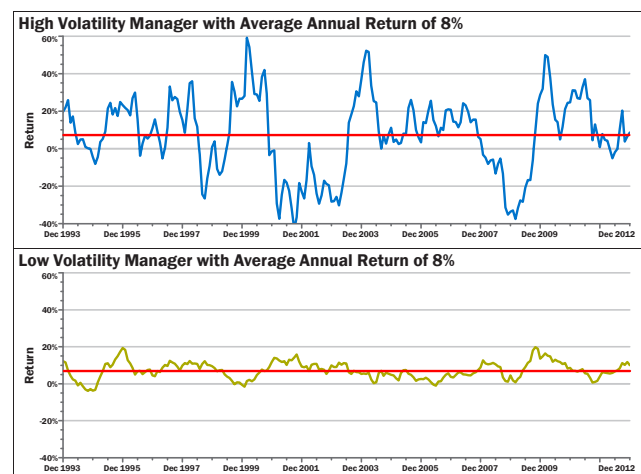
Generally speaking, a lower standard deviation means less uncertainty on a period-to-period basis, which is desirable. The lowest standard deviation possible would be zero. Standard deviations will vary from asset class to asset class, so context is important in distinguishing whether a standard deviation is considered “good” or “bad.”

What Are the Limitations?

Standard deviation does not distinguish between the returns that fall above the average and below the average, so a manager is punished equally for “good” upside deviation and “bad” downside deviation. Also, standard deviation makes no provision for the timing of returns. There is no distinction between a situation where the bad returns were randomly scattered over a long time frame or a scenario when all the bad returns occurred within a small time frame.

What Do the Graphs Show Me?

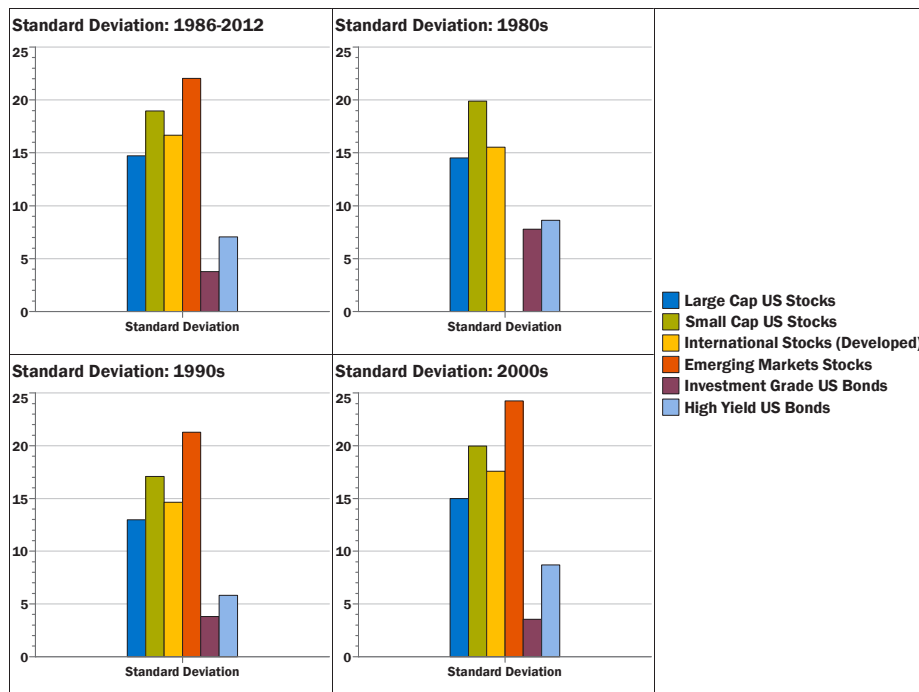
Below we see two return series. The upper one exhibits a high standard deviation and the bottom one shows a low standard deviation. The straight red line represents the long-term, average annual return for each series. As it turns out, the long-term, average returns are identical. However, on a month-to-month basis, the upper graph’s returns tend to stray further from the red line. Therefore, the fund in the upper graph exhibits greater volatility, larger standard deviation, and more substantial risk.



Standard Deviation

What Are Typical Values?

One of the striking aspects of standard deviation is that there is surprisingly little variation across decades. From a returns perspective, the difference between the bull markets of the 1980s and 1990s and the bear markets of the 2000s was extreme. However, looking at the standard deviations across decades, the numbers remain somewhat stable. Overall bonds exhibit the lowest volatility, while emerging markets and small cap stocks have displayed the highest.



January 1986 - December 2012

Standard Deviation	1980s	1990s	2000s	Common 1/86 - 12/12
Large Cap US Stocks	14.52%	12.98%	14.99%	14.73%
Small Cap US Stocks	19.88%	17.08%	19.97%	18.96%
International Stocks (Developed)	15.53%	14.63%	17.58%	16.66%
Emerging Markets Stocks	N/A	21.28%	24.24%	22.04%
Investment Grade US Bonds	7.80%	3.80%	3.53%	3.77%
High Yield US Bonds	8.63%	5.82%	8.71%	7.06%

Related Metrics

Sharpe Ratio: the trade-off of return per unit of volatility risk

Downside Deviation: the amount of “bad” volatility risk

Upside Deviation: the amount of “good” volatility risk

Math Corner

Standard deviation is a well-known statistical tool used across many industries in order to determine just how representative the mean value of an overall set of data is. The process of squaring the differences is used to remove negative values. Otherwise, the positive and negative values would net out to zero.

$$St\ Dev (r_1, \dots, r_n) = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (r_i - \bar{r})^2}$$

Tracking Error

Also known as the standard deviation of excess returns, tracking error measures how consistently a manager outperforms or underperforms the benchmark.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK		TRACKING ERROR		
TRADE-OFF				

How Is it Useful?

Tracking error measures the consistency of excess returns. It is created by taking the difference between the manager return and the benchmark return every month or quarter and then calculating how volatile that difference is. Tracking error is also useful in determining just how “active” a manager’s strategy is. The lower the tracking error, the closer the manager follows the benchmark. The higher the tracking error, the more the manager deviates from the benchmark.

What Is a Good Number?

A “good” tracking error depends upon investor preference. If the investor believes markets are efficient and that it is difficult for active managers to consistently add value, then that investor would prefer a lower tracking error. Alternatively, if the investor believes that smart active managers can add significant value and should not be “tied down” to a benchmark, the investor would tolerate higher levels of tracking error.

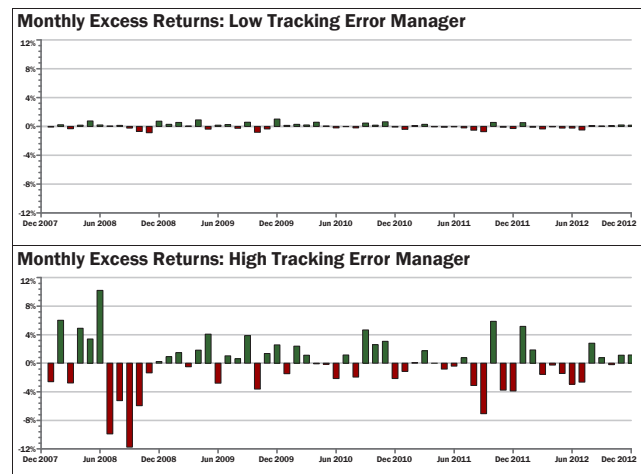
What Are the Limitations?

Tracking error cuts both ways, measuring both periods of outperformance and underperformance versus the benchmark. An investor would prefer high tracking error if there was a high degree of outperformance, but a low tracking error if there was consistent underperformance. Tracking error does not distinguish between the two.

What Do the Graphs Show Me?

Below are two very different active managers. The green bars represent months of outperformance. The red bars are months of underperformance versus the benchmark. Tracking error is created by taking the standard deviation of the red and green bars.

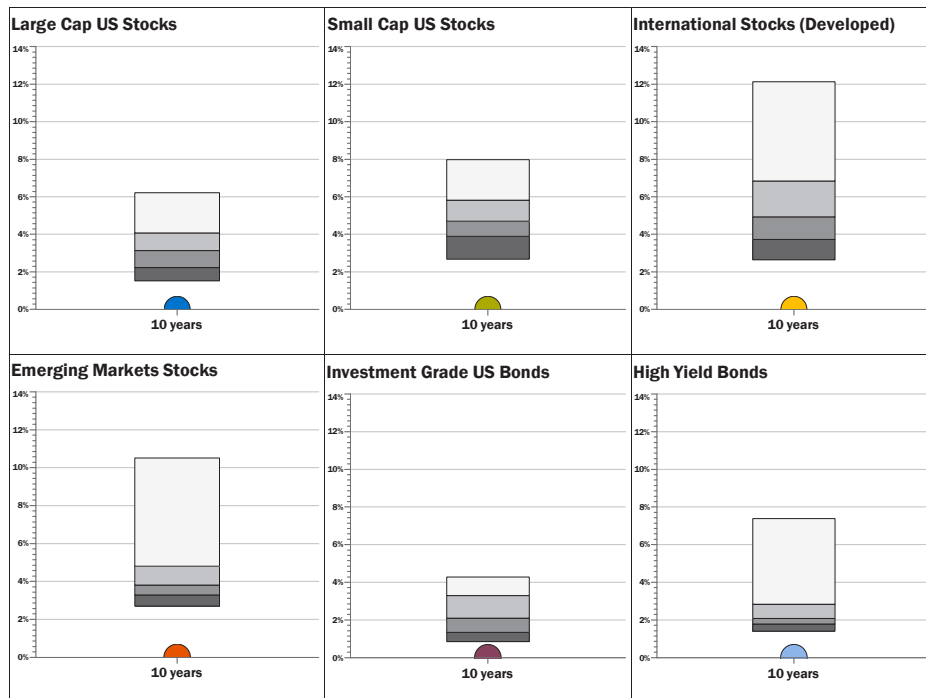
We can infer just how active a manager’s strategy is from the below Information. The small performance deviations seen in the upper graph likely indicate the manager is only making small bets away from the benchmark. However, in order to generate the large monthly performance differentials (for better or worse) in the lower graph, the manager is likely taking big, active bets away from the benchmark.



Tracking Error

What Are Typical Values?

There isn't a typical value for tracking error. Instead, there is a wide spectrum of products available in every asset class, ranging from purely passive to very active. Theoretically, an index fund should have a tracking error of zero relative to its benchmark. Enhanced index funds typically have tracking errors in the 1%-2% range. Most traditional active managers have tracking errors around 4%-7%. Those active managers who are willing to take bigger bets away from an index might exhibit tracking errors in the 10%-15% range. Absolute return, benchmark-agnostic strategies could have even higher tracking errors.



January 2003 - December 2012 • Symbol = Benchmark Index

Tracking Error Funds In the Universe	Large Cap 230	Small Cap 94	International 325	Emerging 64	Gov/Corp 293	HY Bond 96
5th Percentile	1.52%	2.68%	2.63%	2.70%	0.71%	1.41%
25th Percentile	2.22%	3.89%	3.73%	3.28%	0.93%	1.79%
Median	3.13%	4.71%	4.92%	3.81%	1.23%	2.07%
75th Percentile	4.08%	5.82%	6.84%	4.80%	1.84%	2.83%
95th Percentile	6.21%	7.98%	12.12%	10.51%	3.47%	7.38%

Related Metrics

Excess Return: the difference between a manager's returns and the benchmark's returns

Information Ratio: a manager's added value and consistency of added value

Standard Deviation: the degree to which individual returns diverge from the average return

Math Corner

Calculating tracking error is a three-step process. First, an excess return series is created by calculating the periodic differences between the manager and the benchmark. Next, the mean of that excess return series is calculated. Finally, the dispersion of individual observations from the mean excess return is calculated.

$$\text{Tracking Error} = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^n (\text{exc. rtn}_i - \overline{\text{exc. rtn}})^2}$$

Treynor Ratio

A return-versus-risk trade-off metric, the Treynor ratio measures the added value per unit of market risk, with beta defined as risk.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				
TRADE-OFF		TREYNOR RATIO		

How Is it Useful?

The Treynor ratio is similar to the Sharpe ratio. In both cases, the measure of return is the excess over the risk-free investment. The two differ in their definitions of risk. The Sharpe ratio uses standard deviation to define volatility risk, whereas the Treynor ratio uses beta as a measure of market or systematic risk. The Treynor ratio is useful in determining how a particular investment contributes to a diversified portfolio.

What Is a Good Number?

Generally speaking, the higher the Treynor ratio, the better. One hopes the excess return over the risk-free rate is large. However, one should be cautious of Treynor ratios that appear abnormally high. If a Treynor ratio is too large, it could be the result of the beta in the denominator being very small. Such a scenario might indicate an incorrectly specified benchmark.

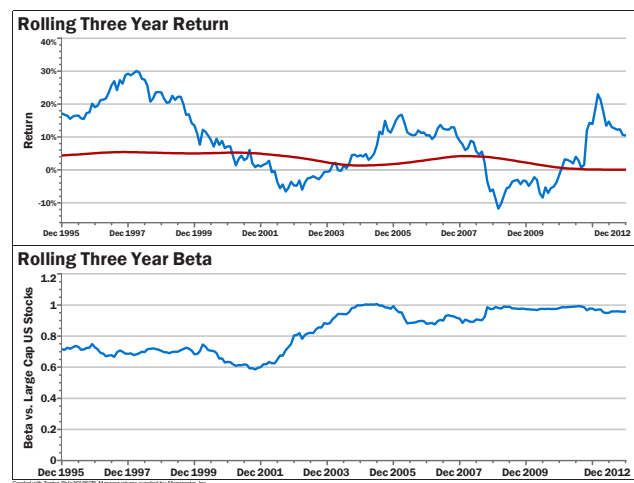
What Are the Limitations?

Like all benchmark-relative metrics, choosing the proper benchmark as a reference point is key. However, this creates a bit of a quandary if the Treynor ratio seeks to measure how the addition of the investment impacts the risk of the portfolio. If one has a broadly diversified portfolio covering many different asset classes, what would the appropriate benchmark be for the portfolio?

Perhaps the best use for the Treynor ratio lies in evaluating a multi-manager line-up. If the investor is planning to have several managers representing each of the major asset classes, then one can first specify the correct benchmark for each given asset class, then calculate Treynor ratios for each of the managers within each asset class, and then examine how different combinations of similar managers work together.

What Do the Graphs Show Me?

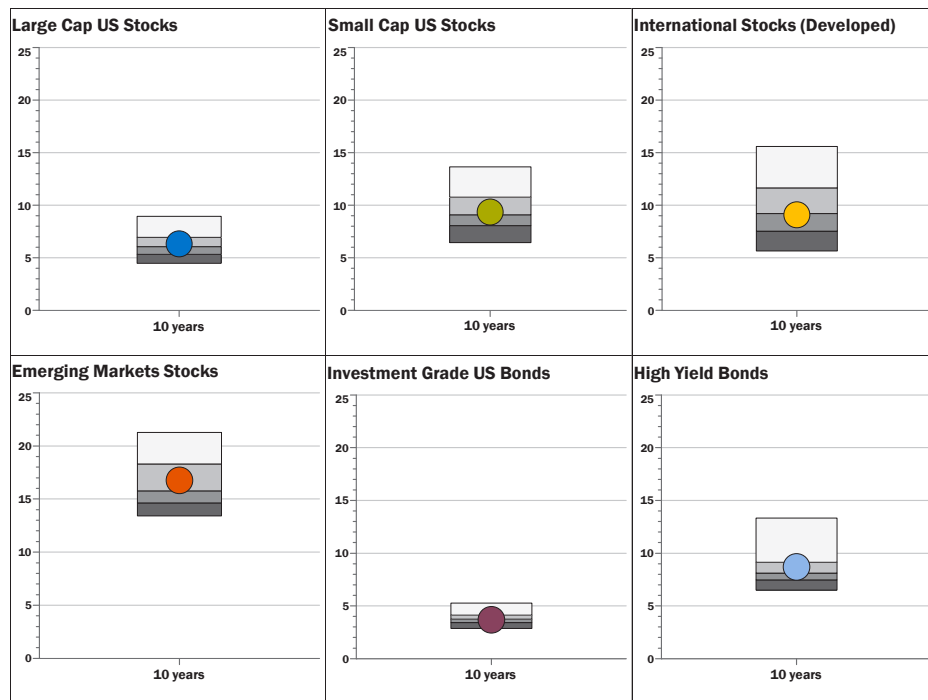
Below are graphs representing the two halves of the Treynor ratio. The upper graph shows the rolling excess return above the risk-free rate. The lower graph shows the risk metric, which is the rolling-period beta versus the appropriate benchmark. The Treynor ratio rolls both of these measures into a single metric. However, one should be aware of the impact that low betas might have on the calculation of a Treynor ratio. If the benchmark chosen isn't a good fit to the manager, the beta might be low, resulting in a misleadingly high Treynor ratio.



Treynor Ratio

What Are Typical Values?

The table to the right displays 10-year Treynor ratios for separately managed account composites covering six asset classes, calculated relative to the appropriate benchmarks for each category. Across all six asset classes, the median manager exhibits roughly the same Treynor ratio as the benchmark. Overall, the Treynor ratios are highest for emerging markets, as emerging markets have had the best performance relative to the risk-free rate over the last 10 years with modest ranges of beta (see also the beta StatFACTS for ranges).



January 2003 - December 2012 • Symbol = Benchmark Index

Treynor Ratio Funds In the Universe	Large Cap 230	Small Cap 94	International 325	Emerging 64	Gov/Corp 293	HY Bond 96
5th Percentile	8.96	13.65	13.82	21.28	5.06	13.32
25th Percentile	6.95	10.76	10.34	18.29	4.10	9.13
Median	6.06	9.10	8.64	15.76	3.75	8.12
75th Percentile	5.32	8.06	7.31	14.64	3.47	7.47
95th Percentile	4.47	6.45	5.43	13.42	2.92	6.48
Index	6.34	9.37	9.10	16.76	3.73	8.69

Related Metrics

Beta: the sensitivity of a manager to a benchmark

Sharpe Ratio: the trade-off of return per unit of volatility risk

Math Corner

The Treynor ratio actually pre-dates its more famous cousin, the Sharpe ratio. The first version appeared in early 1965 in the Harvard Business Review under the title "How to Rate Management of Investment Funds." Treynor originally wanted to examine portfolio performance with the market impact neutralized. Eventually the formula below became that standard definition of Treynor ratio.

$$\text{Treynor Ratio} = \frac{\text{AnnRtn}(r_1, \dots, r_n) - \text{AnnRtn}(c_1, \dots, c_n)}{\text{Beta of mgr to BM}}$$

Up/Down Capture

Up capture measures the percentage of market gains captured by a manager when markets are up. Down capture measures the percentage of market losses endured by a manager when markets are down.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN		UP CAPTURE		
RISK		DOWN CAPTURE		
TRADE-OFF				

How Is it Useful?

It's said that the two elements that drive the market are "greed" and "fear." The up capture and down capture ratios are a useful way of separating the two so they can be analyzed independently. Also, up capture and down capture address the shortcomings of beta, which fails to distinguish between up and down markets.

What Is a Good Number?

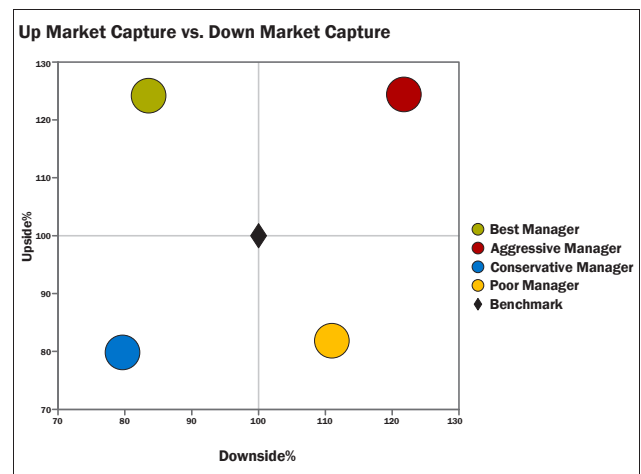
The up capture ratio should be greater than 100%, which would indicate that during periods when the market is up, the manager, on average, did even better. The higher the up capture, the better. Alternatively, down capture ratios should be less than 100%, meaning that when the market went down the manager caught only a fraction of the losses. The lower the down capture, the better. Although rare, it is possible to see negative down captures, indicating that when markets are down the manager tends to be up.

What Are the Limitations?

Up capture and down capture are more useful over longer periods of time. If an asset class has only a few down periods over a short period of time (e.g. short-term investment grade bonds), down capture might not yield very useful information.

What Does the Graph Show Me?

The graph below represents the typical display of up and down capture. The reference point is the benchmark, as noted by the crosshairs in the middle. The top left quadrant represents the ideal location. Here, the green manager is up more than the market in up periods and down less in falling markets. The blue manager lies in the conservative lower left section. This manager lags when markets are up, but hedges in down markets. The red manager in the top right stands in the aggressive quadrant, riding high in up markets but losing more in down markets. The yellow manager languishes in the least desirable quadrant, lagging in up markets and losing more in down markets.



Up/Down Capture

What Are Typical Values?

The tables to the right illustrate the ranges of 10-year up capture and down capture metrics for separately managed account composites across six asset classes. During both up and down markets, about half of the managers in each category (i.e. the 25th to 75th percentiles) tend to capture 90% to 110% of the market movements, with the median near 100%. The conclusion that can be drawn is that most managers track the market in both directions fairly closely. The down capture ratios for the investment grade US bond universe might appear a bit odd, but can be explained by the fact that losses in that asset class are rarer than in equities.

Up Capture Funds In the Universe	Large Cap 230	Small Cap 94	International 325	Emerging 64	Gov/Corp 293	HY Bond 96
5th Percentile	116.62%	110.74%	126.88%	111.05%	128.53%	130.81%
25th Percentile	105.92%	102.98%	109.58%	105.27%	110.42%	109.79%
Median	101.69%	98.51%	100.75%	101.42%	103.21%	100.12%
75th Percentile	96.19%	93.37%	94.42%	97.73%	91.11%	93.05%
95th Percentile	79.72%	82.45%	75.00%	89.94%	52.22%	77.01%

Down Capture Funds In the Universe	Large Cap 230	Small Cap 94	International 325	Emerging 64	Gov/Corp 293	HY Bond 96
5th Percentile	81.69%	80.17%	78.95%	85.16%	24.17%	44.57%
25th Percentile	97.42%	93.28%	95.88%	97.97%	87.87%	91.96%
Median	102.72%	99.03%	102.56%	102.71%	100.71%	104.24%
75th Percentile	105.87%	104.55%	107.92%	105.30%	115.62%	113.42%
95th Percentile	111.79%	109.09%	115.68%	109.80%	147.44%	136.02%

January 2003 - December 2012

Related Metrics

Beta: the sensitivity of a manager to a benchmark

Excess Return: the difference between a manager's returns and the benchmark's returns

Alpha: a measure of "manager skill," adjusted for the level of market risk

Math Corner

For up capture, the first step identifies all the periods in which the market was up. For those up-market periods, the returns for both the manager and the benchmark are geometrically compounded and then annualized. Finally, a ratio between the two is calculated.

$$Up\ Capture = \frac{(\prod_{i=1}^{n_{up}} 1 + r_i)^{1/y} - 1}{(\prod_{k=1}^{n_{up}} 1 + BM_k)^{1/y} - 1}$$

The down capture process is the same, but for down market periods.

$$Down\ Capture = \frac{(\prod_{i=1}^{n_{down}} 1 + r_i)^{1/y} - 1}{(\prod_{k=1}^{n_{down}} 1 + BM_k)^{1/y} - 1}$$

It is possible for the manager to have a negative return in a period when the market is up. By the same token, it is possible for the manager to have a positive return during a period when the market is down. Obviously, the latter is preferred to the former.

It is also worth noting that up capture and down capture values can differ significantly if the underlying periods used in the calculation are monthly or quarterly. For example, assume that over the course of a quarter the monthly returns were -1.2%, +5.2%, and -0.8%. Compounded, the quarterly return was +3.1%. Using monthly data would result in one up period and two down periods. However, using quarterly data in the calculation of the capture ratios results in one up period and no down periods.

Upside/Downside Omega

Upside omega measures the count and scale of returns above a minimum acceptable return (MAR). Downside omega measures the count and scale of returns below the MAR.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				UPSIDE OMEGA
RISK				DOWNSIDE OMEGA
TRADE-OFF				

How Is it Useful?

Upside omega and downside omega are simply the numerator and denominator of the omega ratio separated into individual parts. While the omega ratio is useful for quantifying the trade-off between upside gains and downside losses, sometimes the details get lost because both the good and the bad are rolled into one number. By breaking omega into its constituent parts, one can focus only on the return or only on the risk element.

What Is a Good Number?

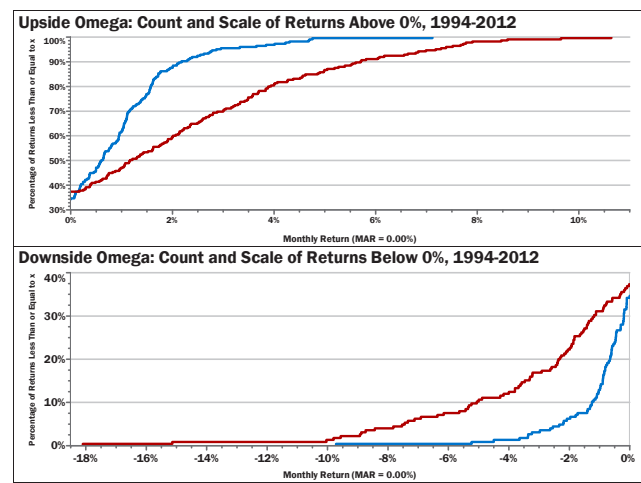
One would want the upside omega to be large, indicating 1) a large number of gains, 2) gains far above and beyond the MAR, or 3) a combination of the two. By the same token, downside omega is hoped to be small, indicating 1) few returns below the MAR, 2) losses not too far below the MAR, or 3) a combination of the two. One should keep both the time frame and the asset class being observed under consideration.

What Are the Limitations?

The values associated with upside and downside omega are abstractions. Taken as stand-alone numbers, it is difficult to understand what they mean. It is almost necessary to visualize the charts to the right, or on the reverse side of this page, in order to grasp the meaning of the upside and downside omega.

What Do the Graphs Show Me?

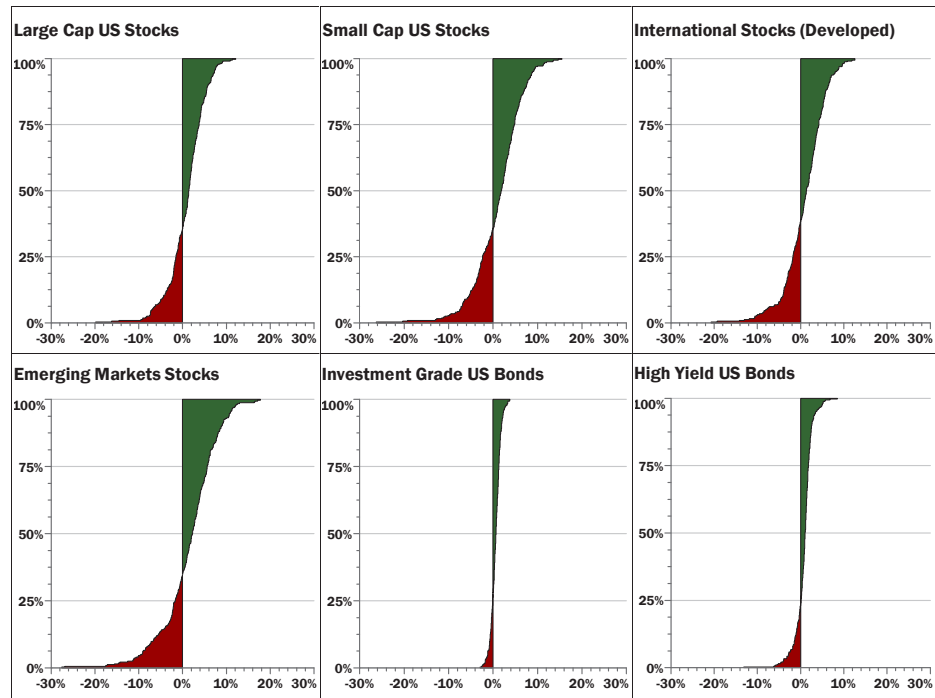
The upper graph shows the count and scale of observations above the MAR, which is the upside omega. The lower graph shows the count and scale of observations below the MAR, which is the downside omega. The graphs feature two investments, an aggressive manager in red and a conservative manager in blue. The aggressive red manager displays substantial gains coupled with a large number of losses. The conservative investment in blue differs substantially. It shows limited gains, but also protects well on the downside. Omega rolls both of these numbers into a single ratio.



Upside/ Downside Omega

What Are Typical Values?

The numbers in the table to the right illustrate the usefulness of breaking omega into its upside and downside components. If one were to look only at the omega ratio in aggregate, the typical values for equity asset classes aren't well differentiated. Most values fall in the 1.7 to 1.9 range. The omegas for large cap US stocks and emerging markets are close to identical. However, by looking at upside omega and downside omega separately, one can see that upside and downside areas are roughly half again as large for emerging markets as they are for large cap US stocks.



January 1986 - December 2012

	Upside Omega (MAR= 0.00%)	Downside Omega (MAR= 0.00%)	Omega (MAR= 0.00%)
Large Cap US Stocks	2.18%	1.22%	1.80
Small Cap US Stocks	2.82%	1.59%	1.77
International Stocks (Developed)	2.42%	1.40%	1.72
Emerging Markets Stocks	3.34%	1.74%	1.92
Investment Grade US Bonds	0.81%	0.20%	4.14
High Yield US Bonds	1.24%	0.42%	2.96

Related Metrics

Omega: the count and scale of returns above versus below a breakpoint

Skewness: a measure of the direction and degree that a set of returns is tilted by its outliers

Kurtosis: an indicator of where the volatility of an investment came from

Math Corner

The upside omega is an integral, defined by the minimum acceptable return (MAR) on one axis and the count and scale of observations above the MAR at the upper bound. Downside omega is the same, but counting the observations below the MAR.

$$\text{Upside Omega} = \sum_{r_i > \text{MAR}} (r_i - \text{MAR})$$

$$\text{Downside Omega} = \sum_{r_i < \text{MAR}} (\text{MAR} - r_i)$$

Value at Risk

A tail risk metric, Value at Risk (VaR) quantifies the amount of expected loss under rare-but-extreme market conditions.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				VALUE AT RISK
TRADE-OFF				

How Is it Useful?

Markets experience losses, and occasionally those losses are extreme. Investors should be financially and mentally prepared to deal with the outcomes of these rare, but traumatic, events. Value at Risk describes how much is typically lost in a day, month, or quarter when markets are at their worst.

What Is a Good Number?

Since VaR is a risk metric measuring loss, the smaller the VaR, the better. Ideally, the VaR would be 0.0%, but no investment carries zero risk. Therefore, it is up to the investor to ask what level of occasional loss would be acceptable to bear.

What Are the Limitations?

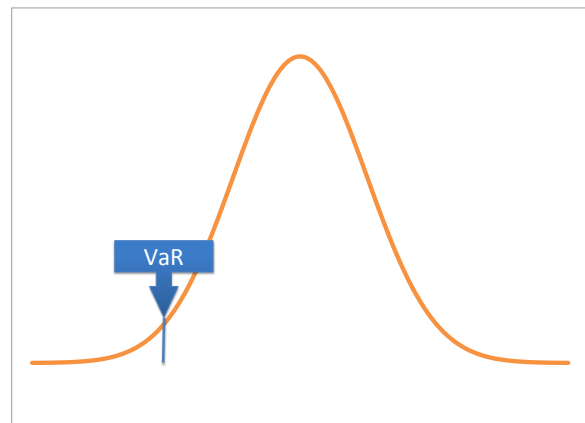
It is important to remember that VaR does not represent the maximum amount one can possibly lose. After all, the most one could potentially lose is 100% of an investment. VaR is not an absolute number. Rather, VaR represents a breakpoint that is exceeded only under extreme conditions.

In addition, VaR does not take into account the upside potential of an investment. Often times, those asset classes or investments with the highest potential for loss, also offer the greatest potential for gain.

What Do the Graphs Show Me?

The graph below illustrates an idealized distribution curve of returns. Most of the time, an investment's returns occur near the center or "peak" of the distribution. When markets are doing very well, the returns will fall to the far right of the curve. However, at other times, the returns will fall to the left or far left of the distribution. Value at Risk is calculated by placing a cut-off point on this part of the curve.

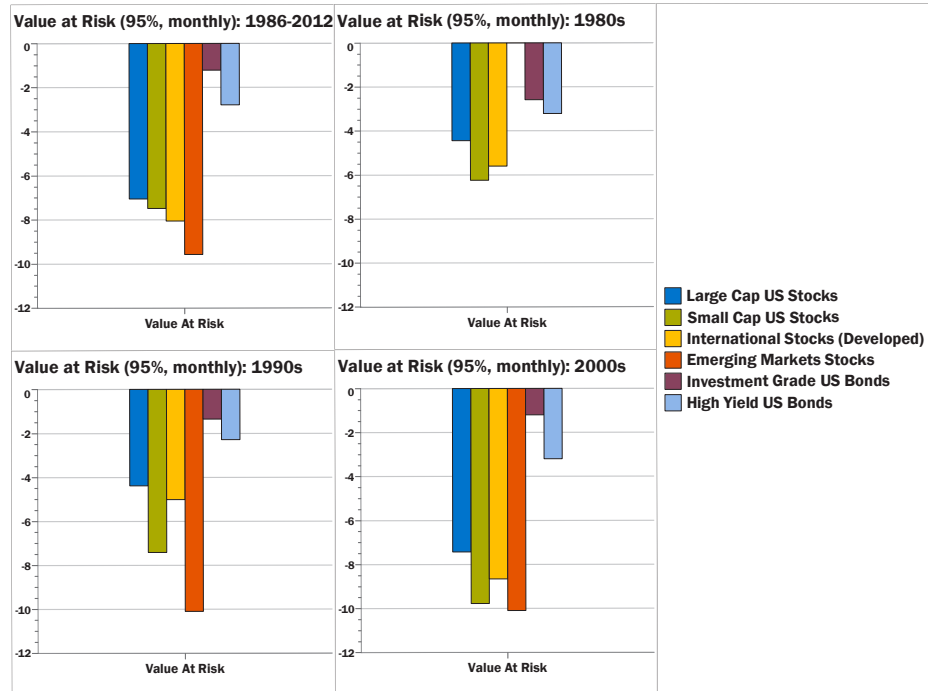
There are many different ways to calculate Value at Risk. Two major inputs to the calculation are the shape of the distribution and where to place the cut-off point. In the real world, most return distributions are not as smooth as the one shown below. Different mathematical solutions accommodate the shape of real world data. Finally, the analyst must determine the appropriate location of the VaR cut-off point. Usually, cut-offs are set between 95% and 99%.



Value at Risk

What Are Typical Values?

Context is all-important when analyzing Value at Risk. One must take the asset class and the underlying time period into consideration when evaluating an investment's VaR. To the right are VaRs calculated at a 95% cut-off level. Not surprisingly, equities have had steeper VaRs than fixed income, with emerging market stocks being the worst. In addition, the VaRs tended to be much worse in the 2000s. While the 1980s and 1990s are remembered as bull markets, the 2000s started and ended with two significant bear markets.



January 1986 - December 2012

Value at Risk	1980s	1990s	2000s	Common 1/86 - 12/12
Large Cap US Stocks	-4.44%	-4.39%	-7.43%	-7.05%
Small Cap US Stocks	-6.25%	-7.42%	-9.77%	-7.48%
International Stocks (Developed)	-5.60%	-5.01%	-8.66%	-8.06%
Emerging Markets Stocks	N/A	-10.10%	-10.09%	-9.57%
Investment Grade US Bonds	-2.58%	-1.35%	-1.20%	-1.21%
High Yield US Bonds	-3.21%	-2.28%	-3.20%	-2.79%

Note: Emerging market data is incomplete for the 1980s, therefore the value is N/A.

Related Metrics

Conditional Value at Risk (CVaR): the average of the losses that fall beyond VaR

Pain Index: the depth, duration, and frequency of losses

Maximum Drawdown: the largest peak-to-trough losses

Downside Omega: the count and scale of returns below a breakpoint

Math Corner

The general equation for Value at Risk can be stated as:

$$1 - c = \int_{-1}^{VaR} p(x) dx$$

Calculating Value at Risk requires different assumptions about the variables in the above equation. For example, "c" represents the cut-off point along the distribution curve where one sets the VaR breakpoint. Values typically fall between 95% to 99%.

The $p(x)dx$ term is the probability density of getting a return with value "x". It addresses the shape of the distribution of returns. Zephyr StyleADVISOR provides two options in defining the distribution. The first is to use a non-parametric distribution, where the historical data is assumed to be representative of all possible outcomes. While trivial to calculate, it requires a large amount of data in order to be considered robust.

The second option is to use a Cornish-Fisher distribution, which assumes the distribution is close to the classic, normal distribution, but does have some amount of skewness and kurtosis. Cornish-Fisher presents a better alternative with smaller data sets. However, it does not work well if the data has large degrees of skewness or kurtosis.

Zephyr K-Ratio

A return-versus-risk statistic, the Zephyr K-ratio measures the rate at which wealth is created and the consistency of the path of wealth creation.

StatMAP

	VOLATILITY	BENCHMARK	CAPITAL PRESERVATION	TAIL
RETURN				
RISK				
TRADE-OFF	ZEPHYR K-RATIO			

How Is it Useful?

The Zephyr K-ratio answers two questions many investors care about most, “At what rate did I grow my wealth?” and “Was that growth of wealth consistent?”

What Is a Good Number?

With the Zephyr K-ratio, a high numerator indicates a high rate of wealth creation. A low denominator indicates consistency in that rate of appreciation. Roll those two goals together and you would hope to see a high Zephyr K-ratio.

However, there is no concrete dividing line above which one can say the ratio is “good” or “bad.” One must compare the Zephyr K-ratio of a manager to the Zephyr K-ratios of its peers, or of an index to get a feel for whether or not it is relatively better or worse than the alternatives.

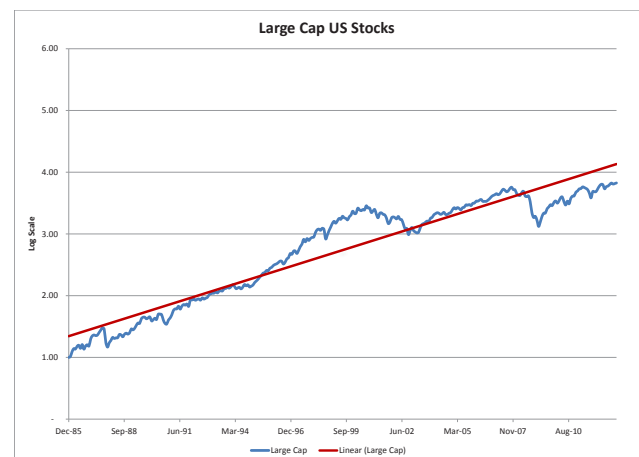
What Are the Limitations?

The Zephyr K-ratio quantifies wealth creation and consistency using methods more commonly found in pure statistical analysis. The mathematical techniques behind the Zephyr K-ratio are well-established and frequently used in the world of statistics. However, it tends to be difficult to easily explain the inner workings of the calculation.

What Does the Graph Show Me?

The blue line that zigs and zags is the cumulative return of a manager, while the red line is a best-fit straight line superimposed over the actual data series. The slope of that best-fit line is the rate of appreciation of wealth and is the numerator of the Zephyr K-ratio. The steeper the slope, the greater the appreciation of wealth. The investor would hope the straight line is sloped upwards as steeply as possible.

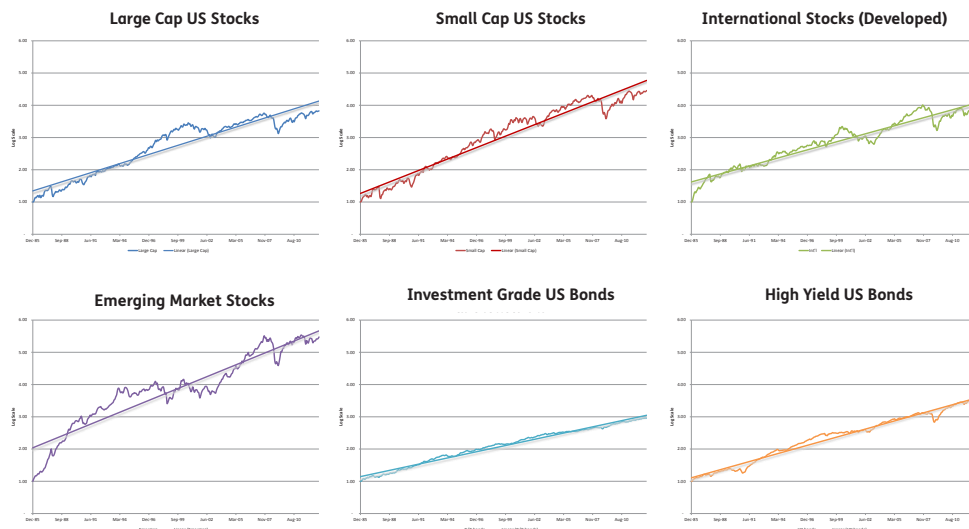
Risk is defined by how much the actual blue line of returns strays from the idealized, straight and narrow red line of consistent wealth creation. The statistical term for this is the standard error of the slope, and it is the denominator of the Zephyr K-ratio. The larger this number, the more inconsistent the wealth creation.



Zephyr K-Ratio

What Are Typical Values?

The values typically seen for Zephyr K-ratios will vary widely depending upon two main factors, the asset class and the time frame being analyzed. Looking at asset classes over long time horizons, we see that investment grade fixed income investments exhibited the highest Zephyr K-ratios. This is driven more by the consistency of the wealth creation, rather than the overall, absolute return. Those asset classes that are more momentum-driven, like emerging markets, tend to have lower Zephyr K-ratios. While emerging markets have offered long-term wealth creation, the fact that the returns are so “feast or famine” leads to lower Zephyr K-ratios.



January 1986 - December 2012

Zephyr K-Ratio	1980s	1990s	2000s	Common 1/86 - 12/12
Large Cap US Stocks	66.13	61.82	6.72	56.23
Small Cap US Stocks	49.06	65.36	12.99	80.27
International Stocks (Developed)	38.08	45.56	11.69	67.86
Emerging Markets Stocks	N/A	16.47	20.55	51.31
Investment Grade US Bonds	68.07	69.48	52.43	142.99
High Yield US Bonds	61.12	64.37	28.32	118.48

Related Metrics

Sharpe Ratio: the trade-off of return per unit of volatility risk

Math Corner

The original variant of what would eventually become the Zephyr K-ratio was proposed by Lars Kestner in 1996 using well-established statistical theories. The summary formula comparing the rate of wealth appreciation against the consistency of wealth appreciation is:

$$\text{slope} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

The standard error of the slope, the denominator of the Zephyr K-ratio, can be calculated from the x- and y-values by the formula.

$$\text{Zephyr K - Ratio} = \frac{\text{slope}}{\text{standard error of slope}}$$

To calculate the Zephyr K-ratio, one should first replace the dates on the horizontal axis of the portfolio's cumulative return graph with consecutive integers starting at 0. With these integers as independent x-values and the corresponding cumulative return values as dependent y-values, one can now calculate the slope of the regression line, the numerator of the Zephyr K-ratio, by the well-known formula.

$$\text{standard error of slope} = \sqrt{\frac{\sum_{i=1}^n (y_i - \bar{y})^2 - \frac{[\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})]^2}{\sum_{i=1}^n (x_i - \bar{x})^2}}{(n-2) \cdot \sum_{i=1}^n (x_i - \bar{x})^2}}$$